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à la mémoire de Nicolas Bergeron

SOUTIENS FINANCIERS

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CICMA (Centre Interuniversitaire en Calcul Mathématique Algébrique)
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Exposés /Talks

Below are to be found the titles and abstracts of the talks of the (preliminary) program. They are listed in alphabetic order according to the first letter of the last name of the speaker which has been submitted on the google sheet registration form.

Résumés / Abstracts

Rebecca Abi Abdallah [Student] (Université de Montréal): *Divisors that are shifted primes*

ABSTRACT. Erdős' 1955 multiplication table problem asks how many distinct products ab there are for $a, b \leq N$. This problem is related to counting the number of integers less than x that have a divisor in the interval $(y, z]$. In this talk, we focus on a variant of the latter quantity, where the divisor is restricted to being a shifted prime, an integer of the form $p-1$, with p prime. The study of its asymptotic behaviour for different values of z reveals a phase transition at $z = y \cdot \exp\left((\log y)^{2-\log 4}\right)$, highlighting the interdependent behaviour of these divisors. This result expands Ford's work, where he established the order of magnitude of the quantity in the case where $z = \infty$. This is joint work with Dimitris Koukoulopoulos, Valeriya Kovaleva, Jeremy Schlitt and Neo Tardy.

Niven Achenjang [Student] (MIT): *On Brauer groups of stacky curves*

ABSTRACT. Brauer groups $\mathrm{Br}(\_) = \mathrm{H}^2(\_, \mathbb{G}_m)$ of fields and schemes have found much utility in number theory, but Brauer groups of stacks have only relatively recently started receiving more attention. In this talk, I would like to convince you that interesting new phenomena emerge already for Brauer groups of stacky curves. After mentioning some known computations carried out for a couple specific modular curves, I will sketch a computation of the Brauer groups of quite general stacky curves over algebraically closed fields (spoiler: unlike in the scheme case, these are often nonzero).

Taiga Adachi [Student] (Kyushu University, Japan): *Iwasawa theory for weighted graphs*

ABSTRACT. Recently, Gonet and Vallières initiated Iwasawa theory for weighted graphs. They studied an asymptotic behavior of the numbers of spanning trees of $\mathbb{Z}_p$-covers of graphs as an analogue of classical Iwasawa theory. After them, Iwasawa theory for graphs has been further developed by DuBose, Kataoka, Kleine, Lei, McGown, Müller, Ray, and others. Although these previous works focused on graphs without weights, studying weighted graphs is important for applied mathematics such as quantum walks. In this talk, we generalize Iwasawa theory for graphs to weighted graphs. In particular, we prove an analogue of Iwasawa's class number formula and that of Kida's formula for $\mathbb{Z}_p^d$-covers of weighted graphs. This is joint work with Kosuke Mizuno and Sohei Tateno.

Ömer Avci [Student] (Bogazici University): *Torsion of Rational Elliptic Curves over the Cyclotomic Extensions of* $\mathbb{Q}$

ABSTRACT. Let E be an elliptic curve defined over $\mathbb{Q}$. Let $p > 3$ be a prime such that $p-1$ is not divisible by $3, 4, 5, 7, 11$. In this article we classify the groups that can arise as $E(\mathbb{Q}(\zeta_p))_{\mathrm{tors}}$ up to isomorphism. The method illustrates techniques for eliminating possible structures that can appear as a subgroup of $E(\mathbb{Q}^{ab})_{\mathrm{tors}}$.

Romain Brancherau [Postdoc] (McGill. U.) :*Generating series of modular symbols in* SL_N

ABSTRACT. I will explain a construction of a map from the homology of locally symmetric spaces associated to SL_N, to modular forms of weight N. The image of a cycle C by this map is a modular form whose Fourier coefficients are intersection numbers between C and a family of generalized modular symbols on the locally symmetric space. This map can be seen as a Kudla-Millson theta lift for the dual pair (SL_N, SL_2) and is also related to a construction of Bergeron-Charollois-Garcia.

Elias Caeiro [Student] (École Normale Supérieure): *The class number one problem via p-adic uniformisation of Shimura curves*

Abstract. The class number one problem, solved by Heegner, Baker and Stark, asserts that there are only 9 class number one imaginary quadratic fields. In joint work with Henri Darmon, we give a new proof of the Heegner-Baker-Stark theorem using Heegner points associated with a parametrisation of a well-chosen elliptic curve E by a Shimura curve. The key ingredients are: the Gross-Zagier- Zhang formula, which implies the triviality of these points when E has rank 2, and the Cerednik-Drinfeld p-adic uniformisation of Shimura curves.

Jérémy Champagne [Student] (University of Waterloo) : Weyl's equidistribution theorem in function fields

Abstract. Finding a proper function field analogue to Weyl's theorem on the equidistribution of polynomial sequences is a problem that was originally considered by Carlitz in 1952. As noted by Carlitz, Weyl's classical differencing methods can only handle polynomials with degree less than the characteristic of the field. In this talk, we discuss some recent methods which avoid this "characteristic barrier", and we show the existence of polynomials with extremal equidstributive behaviour.

This is joint work with Yu-Ru Liu, Thái Hoàng Lê and Trevor D. Wooley.

Pierre Charollois [Faculty] (Sorbonne Université): *Dedekind sums for $SL_3(\mathbb{Z})$ and elliptic units for complex cubic fields*

Abstract. I will present two interrelated recent joint works with Nicolas Bergeron and Luis Garcia. In the first one, we study certain generalized Dedekind sums arising from a 2-cocycle for $SL_3(\mathbb{Z})$. In the second, we propose a conjecture extending the classical construction of CM elliptic units to complex cubic fields by means of special values of the elliptic Gamma function, a youthful special function originating from mathematical physics. We provide both numerical and theoretical evidence supporting our belief that the elliptic Gamma function will play an important role towards the solution of Hilbert's 12th problem.

Timothy Cheek Student and Marie-Helene Tome [Students] (University of Michigan, Duke U.): *On the Density of Low-Lying Zeros of a Large Family of Automorphic L-functions (Talk 1)*

Abstract. Under the generalized Riemann Hypothesis (GRH), Baluyot, Chandee, and Li provided some of the strongest evidence to date on the Katz-Sarnak Density conjecture. They doubled the range in which the 1-level density of low lying zeros of a family of cuspidal forms agrees with Random Matrix Theory. We generalize their techniques to the study of higher centered moments of this family, leading to better results on the behavior near the central point. Numerous technical obstructions emerge that are not present in the one-level density. Averaging over the level of the forms and assuming GRH, we prove the density predicted by Katz and Sarnak holds for the n-th centered moments for test functions whose Fourier transform is compactly supported in $(-\sigma, \sigma)$ for $\sigma = \min\left\{3/2(n-1), 4/(2n - \mathbf{1}_{2\nmid n})\right\}$. For $n = 3$, our results improve the previously best known $\sigma = 2/3$ to $\sigma = 3/4$. We also prove the two-level density agrees with the Katz-Sarnak density conjecture for test functions whose Fourier transform is compactly supported in $\sigma_1 = 3/2$ and $\sigma_2 = 5/6$, respectively, extending the previous best known sum of supports $\sigma_1 + \sigma_2 = 2$. This work is the first evidence of an interesting new phenomenon: by taking different test functions, we are able to extend the range in which the Katz-Sarnak density predictions hold. The techniques we develop can be applied to understanding quantities related to this family containing sums over multiple primes.

Timothy Cheek Student and Marie-Helene Tome [Students] (University of Michigan, Duke U.): *Numerical investigation of lower order biases in moment expansions of one-parameter families of elliptic curves (Talk 2)*

Abstract. For a fixed elliptic curve E without complex multiplication, $a_p =: p + 1 - \#E(\mathbb{F}_p)$ is $O(\sqrt{p})$ and $\frac{a_p}{2\sqrt{p}}$ converges to a semicircular distribution. Michel proved that for a one-parameter family of elliptic curves $y^2 = x^3 + A(T)x + B(T)$ with $A(T), B(T) \in \mathbb{Z}[T]$ and non-constant j-invariant, the second moment of $a_p(t)$ is $p^2 + O(p^{3/2})$. The size and sign of the lower order terms has applications to the distribution of zeros near the central point of Hasse-Weil L-functions and the Birch and Swinnerton-Dyer conjecture. S. J. Miller conjectured that the highest order term of the lower order terms of the second moment that does not average to zero is on average negative. Previous work on the conjecture has been restricted to a small set of highly nongeneric families. We create a database

and a framework to quickly and systematically investigate biases in the second moment of any one-parameter family. When looking at families which have so far been beyond current theory, we find several potential violations of the conjecture for $p \leq 250,000$ and discuss new conjectures motivated by the data.

Raiza Corpuz [Student] (University of Ottawa / University of Waikato): *Equivalences of the Iwasawa Main Conjecture*

ABSTRACT. Let p be an odd prime, and suppose that E_1 and E_2 are two elliptic curves which are congruent modulo p. Fix an Artin representation $\tau : G_F \to \mathrm{GL}_2(\mathbb{C})$ over a totally real field F, induced from a Hecke character over a CM-extension K/F. We compute the variation of the μ- and λ-invariants of the Iwasawa Main Conjecture, as one switches between τ-twists of E_1 and E_2, thereby establishing an analogue of Greenberg and Vatsal's result. Moreover, we show that provided an Euler system exists, $\mathrm{IMC}(E_1, \tau)$ is true if and only if $\mathrm{IMC}(E_2, \tau)$ is true. This is joint work with Daniel Delbourgo from University of Waikato.

Steven Creech [Student] (Brown University): *Mass Equidistribution: Cocompact Vs. Non-Cocompact Surfaces*

ABSTRACT. Given a weight k Hecke-cusp form f on $\mathrm{SL}_2(\mathbb{Z})\backslash\mathbb{H}$, one can define a measure μ_f on the surface. The celebrated mass equidistribution theorem of Helowinsky and Soundararajan states that as $k \to \infty$, the measure μ_f approaches the uniform measure on the surface. Given a cocompact Riemann surface, one can ask the analogous problem, but this remains an open problem. In this talk, I will compare the classical mass equidistribution conjecture to the cocompact case, highlighting where difficulties arise. I will discuss some recent work of Nelson that relates mass equidistribution to a certain kind of shifted convolution sum. Time permitting, I will talk about my current work on the problem suggesting an average statement of the conjecture in the cocompact case.

Mihir Deo [Student] (University of Ottawa): *p-adic Asai L-functions of Bianchi modular forms at non-ordinary primes*

ABSTRACT. Loeffler and Williams constructed p-adic Asai L-functions of Bianchi modular forms, i.e., a p-adic measure that interpolates the critical values of the Asai L-function attached to a p-ordinary Bianchi modular form. In this talk, I will discuss the p-adic Asai L-functions of p-non-ordinary Bianchi modular forms. These p-adic L-functions are not measures, but unbounded p-adic distributions. If time permits, I will also talk about the Lei-Loeffler-Zerbes style decomposition of unbounded p-adic Asai L-functions (distributions) into signed bounded measures.

Julie Desjardins [Faculty] (University of Toronto): *Generalized Eckardt points on del Pezzo surfaces of degree 1*

ABSTRACT. Over an algebraically closed field, del Pezzo surfaces of degree 3 (the smooth cubic surfaces in $\mathbb{P}^3$) contain 27 exceptional curves (which we call lines), and any point on the surface is contained in at most 3 lines; we call such a point an Eckardt point. A del Pezzo surface of degree 2 contains 56 lines, and a point on it is contained in at most 4 lines. For del Pezzo surfaces of degree 1, the situation is a lot more complex. These surfaces contain 240 lines, and Winter and van Luijk proved recently that a point is contained in at most 10 lines outside characteristics 2 and 3. We will call a point contained in 10 lines, outside characteristics 2 and 3, a generalized Eckardt point. Most other questions on the configuration of the lines on a dP1 are still widely unanswered, for instance we don't even know of an example with more than one generalized Eckardt point. In a joint work with K. Isham, Y. Fu and R. Winter, we classify the different ways in which 10 lines can intersect, construct new families of surfaces with a generalized Eckardt point, and discuss strategies for finding more such examples. I will give an overview of these results in this talk.

Joseph DiCapua [Student] (CUNY Graduate Center): *Parametrization of Formal Norm Compatible Sequences*

ABSTRACT. We give a classification of power series parametrizing Lubin-Tate trace compatible sequences. This proof answers a question posed in the literature by Berger and Fourquaux. Lubin-Tate trace compatible sequences are a generalization of norm compatible sequences, which arise in Iwasawa theory and local class field theory. The result we prove generalizes the interpolation theorem proved by Coleman in the classical norm compatible sequence case. We also, jointly with Victor Kolyvagin, give a method for finding such series explicitly in certain special cases.

Nicolas Doyon [Faculty] (Université Laval): *The distribution of the length of minimal addition chains*

ABSTRACT. An addition chain is a sequence of positive integers $1 = a_0, a_1, \ldots, a_k$ such that $a_j = a_s + a_t$ with $s, t < j$ for all j, $1 \leq j \leq k$. A challenging problem is, given a positive integer n, to find the minimal length of an addition chain leading to n which we denote as $\ell(n)$. In this presentation, we will prove bounds on the distribution function of $\ell(n)$ that is on the quantity $H(m,k) := \#\{n < m : \ell(n) < k\}$. Doing so, we precise results obtained by Paul Erdős.

Alessandro Fazzari [Postdoc] (University of Montreal): *Consecutive real quadratic fields with large class numbers*

ABSTRACT. For a given positive integer k, we prove that there are at least $x^{1/2-o(1)}$ integers $d \leq x$ such that the real quadratic fields $\mathbb{Q}(\sqrt{d+1}), \ldots, \mathbb{Q}(\sqrt{d+k})$ have class numbers essentially as large as possible.

Claire Frechette [Postdoc] (Boston College) :*Constructing vector-valued automorphic forms on unitary groups*

ABSTRACT. We introduce a method for producing vector-valued automorphic forms on unitary groups from scalar-valued ones. As an application, we construct an explicit example. Our strategy employs certain differential operators. It is inspired by work of Cléry and van der Geer in the setting of Siegel modular forms, but it also requires overcoming challenges that do not arise in the Siegel setting.

Rylan Gajek-Leonard [Faculty] (Union College): *A bound on μ-invariants of supersingular elliptic curves*

ABSTRACT. Let $E/\mathbb{Q}$ be an elliptic curve and let p be a prime of good supersingular reduction. Attached to E are pairs of Iwasawa invariants $\mu_p^{\pm}$ and $\lambda_p^{\pm}$ which encode arithmetic properties of E along the cyclotomic $\mathbb{Z}_p$-extension of $\mathbb{Q}$. A well-known conjecture of B. Perrin-Riou asserts that $\mu_p^{\pm} = 0$. We provide support for this conjecture by proving that for all but finitely many primes, if $\lambda_p^{\pm} = 0$ then $\mu_p^{\pm}$ is at most 1 or 2, depending on sign.

Louis Gaudet [Postdoc] (University of Massachusetts Amherst): *Sifting for small split primes of an imaginary quadratic field in a given ideal class*

ABSTRACT. Let $D > 3$, $D \equiv 3$ (4) be a prime, and let $\mathcal{C}$ be an ideal class in the field $\mathbb{Q}(\sqrt{-D})$. We give a new proof that $p(D, \mathcal{C})$, the smallest norm of a split prime $\mathfrak{p} \in \mathcal{C}$, satisfies $p(D, \mathcal{C}) \ll D^L$ for some absolute constant L. Our proof is sieve theoretic. In particular, this allows us to avoid the use of log-free zero-density estimates (for class group L-functions) and the repulsion properties of exceptional zeros, two crucial inputs to previous proofs of this result.

Mathilde Gerbelli-Gauthier [Faculty] (University of Toronto): *Growth of Cohomology of Arithmetic Groups*

ABSTRACT. I'll discuss the question of growth of cohomology in towers of arithmetic groups, a topic at the interface of topology and number theory. In particular, I'll describe some results that rely on Hodge-like decompositions of the cohomology coming from the Langlands program to study this problem. This is a topic in which I've learned a lot from Nicolas Bergeron and I hope to highlight his impact on my work throughout the talk. Most of the work discussed is joint with Rahul Dalal.

Wissam Ghantous [Faculty] (University of Central Florida) :*A symmetric p-adic symbol for triples of modular forms*

ABSTRACT. We introduce a new p-adic triple symbol based on the Garrett-Rankin p-adic L-function and show that it satisfies symmetry relations, when permuting the three input modular forms. We also provide computational examples illustrating this symmetry property. To do so, we develop algorithms to compute ordinary projections of nearly overconvergent modular forms as well as certain projections over spaces of non-zero slope. Our work also gives an efficient method to calculate certain Poincaré pairings in higher weight, which may be of independent interest.

Eyal Goren [Faculty] (McGill University): *On generation of lattices by vectors of prescribed norms*

ABSTRACT. Motivated by cryptographic applications, we study the problem of generation of lattices (over totally real fields) by elements of prescribed norm. This is joint work with Jonathan Love.

Tony Haddad [Student] (Université de Montréal): *A coupling for prime factors*

ABSTRACT. Let N_x be an integer chosen uniformly at random from the set $\mathbb{Z} \cap [1, x]$, and let $(V_1, V_2, \ldots)$ be a Poisson–Dirichlet process of parameter 1. We present a coupling of these two random objects satisfying

$$\mathbb{E} \sum_{i \geq 1} |\log P_i - V_i \log x| = O(1)$$

as x grows, with $N_x = P_1 P_2 \cdots$ being the unique factorization of N_x into primes or ones and the P_i's being non-increasing. This establishes a conjecture of Arratia (1998). We also use this coupling to provide a probabilistic proof of the Dirichlet law for the integer factorization into k parts of random integers proved by Leung (2023) and we improve on its error term.

Hazem Hassan [Student] (McGill University): *Integral aspects of Fourier duality for abelian varieties*

ABSTRACT. Classical results on Fourier duality for the Chow rings of abelian varieties give a non-trivial map between the Chow ring of an abelian variety A and its dual. This gives rise to a natural bi-grading on these Chow rings. Further, if one picks a polarization of A, then this machinery leads to an action of $\mathfrak{sl}_2$ on $\mathrm{Ch}(A)$.

These classical results are established for the Chow ring after tensoring with $\mathbb{Q}$. In this talk, I will present integral analogues of these results. This is joint work with Junaid Hasan, Milton Lin, Marcella Manivel, Lily McBeath and Ben Moonen

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Erman Isik [Postdoc] (University of Ottawa): *The growth of Tate–Shafarevich groups of p-supersingular elliptic curves over anticyclotomic $\mathbb{Z}_p$-extensions at inert primes*

ABSTRACT. In this talk, we'll discuss the asymptotic growth of both the Mordell–Weil ranks and the Tate–Shafarevich groups for an elliptic curve E defined over the rational numbers, focusing on its behaviour along the anticyclotomic $\mathbb{Z}_p$-extension of an imaginary quadratic K. Here, p is prime at which E has good supersingular reduction and is inert in K. We will review the definitions and properties of the plus and minus Selmer groups from Iwasawa theory and discuss how these groups can be used to derive arithmetic information about the elliptic curve.

Arihant Jain [Student] (McGill University): *Diagonal restrictions of RMCs and GKZ-type theorems*

ABSTRACT. In recent decades, there has been extensive research on algebraic cycles on Shimura varieties that arise as arithmetic quotients of special orthogonal groups. These orthogonal groups are isometries of real vector spaces of signature (r,2). The recent work of Darmon-Gehrmann-Lipnowski provides a framework for extending some aspects of the arithmetic theory to arbitrary real signatures (r,s). This framework rests on the notion of rigid meromorphic cocycles (RMC) for p-arithmetic subgroups, whose study is initiated in this paper. One of their results is establishing the modularity of a generating series consisting of RMCs. In our talk we plan to prove modularity of some lower rank cases using modularity in higher rank. This is achieved by viewing the lower rank space embedded in the higher rank space. This idea resonates with the work of Zagier where he used Hirzerbruch-Zagier theorem and embeddings of modular curves in Hilbert modular surface to establish Gross-Kohnen-Zagier theorem (GKZ).

Paul Kinlaw [Faculty] (UMaine Presque Isle): *Almost primes of almost prime index*

ABSTRACT. A k-almost prime is a product of k primes, counting repetition. For an increasing sequence $\{a_j\}$ of positive integers, the jth term a_j is called the term of index j. We establish bounds for the counting function of j-almost primes of k-almost prime index for given $j, k \geq 1$. We also establish bounds for the nth j-almost prime of k-almost prime index, as well as the reciprocal sums for given j, k. In particular, we show that for $j = 1$, the reciprocal sum tends to 1 as $k \to \infty$. This is joint work with Megan Triplett and William Tripp.

Andrew Knightly [Faculty] (University of Maine): *Trace formulas for locally supercuspidal newforms*

ABSTRACT. We give a general formula for the trace of a Hecke operator on the span of the weight k newforms whose local components at the ramified places are prescribed supercuspidal representations. To make it completely

explicit, one must compute certain local orbital integrals at the ramified places, and this is carried out when the fixed supercuspidals have conductor exponent 2 or 3. Using this trace formula, we observe bias in global dimension formulas favoring certain supercuspidals over others with the same conductor exponent, though asymptotically there is balance as first shown in a very general setting by Kim, Shin and Templier.

Pavel Coupek [Postdoc] (Michigan State University]): *Ramification bounds via Wach modules and q-crystalline cohomology*

ABSTRACT. Given a Galois representation T obtained as the mod p étale cohomology of a smooth proper variety with good reduction over a p-adic local field K, theory of Breuil-Kisin modules and Breuil-Kisin cohomology can be used to obtain a bound on ramification of T. Under the assumption that K is absolutely unramified, I will describe an alternative approach to obtaining such a bound using Wach modules and q–crystalline cohomology. The resulting bound is stronger than the one obtained via Breuil-Kisin theory, and in partiular, it is able to distinguish the good reduction case from the more general case of semistable reduction.

Valeriya Kovaleva [Postdoc] (Université de Montréal): *Traces of random matrices over finite fields and cancellation in character sums*

ABSTRACT. Let X be a matrix drawn uniformly at random from $GL_n(F_q)$, then one may conjecture that traces of powers of such matrices $Tr(X^k)$ should have an asymptotically uniform distribution F_q. Further, one may wonder how robust this phenomenon is and how large can k be. On the one hand, this question is an analogue of a classic problem from random matrix theory, and, on the other hand, it is intimately related to short character sums over function fields with the power k serving as the conductor. In our work, we prove that the distribution of $Tr(X^k)$ is indeed asymptotically uniform and that the respective short interval character sums exhibit cancellation for $k = q^{o(n^2)}$. This is a much wider range than one could hope to obtain for general characters, and in fact, this phenomenon seems to have no analogue over the integers. This is joint work with Ofir Gorodetsky.

Matilde Lalin [Faculty] (Université de Montréal): *Moments of Artin-Schreier L-functions*

ABSTRACT. We compute moments of L-functions associated to the polynomial family of Artin-Schreier covers over $\mathbb{F}_q$, where q is a power of a prime p, when the size of the finite field is fixed and the genus of the family goes to infinity. More specifically, we compute the kth moment for a large range of values of k, depending on the sizes of p and q. We also compute the second moment in absolute value of the polynomial family, obtaining an exact formula with a lower order term, and confirming the unitary symmetry type of the family. This is joint work with Alexandra Florea and Edna Jones.

Antonio Lei [Faculty] (University of Ottawa): *On Mazur's growth number conjecture*

ABSTRACT. Let E be an elliptic curve defined over $\mathbb{Q}$, K an imaginary quadratic field, and p a prime number. In Iwasawa theory, we are interested in the asymptotic behavior of arithmetic objects attached to E over the $\mathbb{Z}_p^2$-extension of K. In the 1980s, Mazur formulated a conjecture predicting how the Mordell–Weil ranks of E over this extension should behave. In this talk, we discuss several results towards special cases of this conjecture using p-adic methods and Heegner points.

Patrick Letendre [independent researcher] (Unaffiliated): *Divisors and the Number of Divisors Function*

ABSTRACT. Let $\mathcal{D}_n$ denote the set of divisors of an integer n and $\tau(n) = |\mathcal{D}_n|$ denote the number of divisors function. Back in 2015, I gave a talk at the Maine- Québec Number Theory Conference focused on the function $\tau(n)$. The goal was to find the best way to estimate this function given only its size and the value of $\omega(n)$, the number of distinct prime divisors of n. At the beginning of this talk, I will discuss the work I have done since then. I will then address unconventional problems related to the set $\mathcal{D}_n$ as for example the estimate

$$|\{(d_1, d_2, d_3) \in \mathcal{D}_n^3 : d_1 + d_2 = d_3\}| \leq \tau(n)^{2-\delta}$$

with $\delta = 0.045072$.

Sun Kai Leung [Student] (Université de Montréal): *Nonvanishing of Dirichlet L-functions with smooth moduli*

ABSTRACT. In this talk, I will explain how to establish the nonvanishing of central values of primitive Dirichlet L-functions with smooth conductors.

Nikita Lvov [Independent researcher] (Unaffiliated) :*A random walk on the category of finite abelian p-groups*

ABSTRACT. The cokernel of a large p-adic random matrix M is a random abelian p-group. Friedman and Washington showed that its distribution asymptotically tends to the well-known Cohen-Lenstra distribution. We study an irreducible Markov chain on the category of finite abelian p-groups, whose stationary measure is the Cohen-Lenstra distribution. This Markov chain arises when one studies the cokernel of M, after conditioning on a submatrix of M. We show two surprising facts about this Markov chain. Firstly, it is reversible. Hence, one may regard it is a random walk on finite abelian p-groups. The proof of reversibility also explains the appearance of the Cohen-Lenstra distribution in the context of random matrices. Secondly, we can explicitly determine the spectrum of the infinite transition matrix associated to this Markov chain. Finally, the Markov chain gives insight into the universality phenomenon for random p-adic matrices, whose study was initiated by Maples and greatly advanced by Wood and collaborators.

Muhammad Manji [Postdoc] (Concordia U.): *Iwasawa Theory for GU(2,1) at inert primes*

ABSTRACT. We will discuss the construction of some new Selmer groups coming from strange properties of locally $\mathbb{Q}_p^2$-analytic (ϕ, Γ)-modules. These Selmer groups will be used along with a Loeffler-Skinner-Zerbes Euler system and the Schneider-Venjakob locally analytic regulator map to state an Iwasawa main conjecture for $GU(2,1)$ at inert primes.

David Marcil [Student] (Columbia University) :*p-adic L-functions for P-ordinary Hida families on unitary groups*

ABSTRACT. In this talk, I will briefly discuss the notion of P-ordinary automorphic representations on a unitary group, where P is some parabolic subgroup. I will then describe the local structure of such a representation π at p using the theory of types, generalize a certain theorem of Hida, and analyze the P-ordinary Hida family C_π associated to π. Finally, I will briefly introduce a p-adic family of Eisenstein series associated to C_π and explain how to construct a p-adic L-function for C_π using the doubling method, generalizing the one obtained by Eischen-Harris-Li-Skinner in the ordinary setting (i.e. when P is minimal). These results are from the speaker's thesis.

Myrto Mavraki [Faculty] (University of Toronto) :*Bounded geometry for PCF-special subvarieties*

ABSTRACT. A rational map is postcritically finite (PCF) if its critical orbits are finite. Postcritically finite maps play an important role in dynamics. It was suggested by Silverman that PCF points on the moduli space of degree d rational maps M_d play a role analogous to CM elliptic elliptic curves. Inspired in part by the Pink-Zilber conjectures in unlikely intersections, Baker and DeMarco formulated a conjecture aiming to describe the subvarieties of M_d that contain a Zariski dense set of PCF ("special") points. Their conjecture, now known as dynamical André–Oort (or DAO) was recently resolved in the case of curves by Ji–Xie, but remains open in higher dimensions. In this talk we will describe recent work with DeMarco and Ye, providing uniform bounds on the configurations of PCF points in families of subvarieties in M_d. We also provide a gap principle in the spirit of Dimitrov–Gao–Habegger's, Kühne's, and Gao–Ge–Kühne's work on the uniform Mordell–Lang conjecture

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Matt Olechnowicz [Postdoc] (Concordia University): *Merging Beatty sequences*

ABSTRACT. The list of integer parts of the multiples of a positive real number is known as a Beatty sequence. A classical theorem of Rayleigh (1894) states that if α and β are positive irrationals whose reciprocals sum to unity, then the Beatty sequences generated by α and β partition the natural numbers. Since the natural numbers themselves form a Beatty sequence, Rayleigh's theorem gives examples of two Beatty sequences which merge to give a third. In joint work with Jonathan Love, we characterize all such mergers among a natural generalization of Beatty sequences. The results feature surprising connections to the theory of groups, the geometry of lattices, and the number 7.

Tariq Osman [Postdoc] (Brandeis University): *Limit Theorems and Tail Estimates for Higher Rank Theta Sums*

ABSTRACT. We discuss how methods from homogeneous dynamics can be used to study the distribution of values of appropriately normalized exponential sums of the form $\sum_{n\in\mathbb{Z}^2} f(\frac{1}{N}n)e^{2\pi iQ(n)t}$, where t is randomly sampled from the unit interval, and Q is a fixed generic, positive definite, irreducible quadratic form. This is joint work with J. Griffin and J. Marklof.

Jack Petok [Postdoc] (Colby College): *The zeta function of a noncommutative K3 surface*

ABSTRACT. We study the arithmetic of the K3 category associated to a cubic fourfold over a non-algebraically closed field k, specifically, the Galois representation on its ℓ-adic Mukai realization. For k a finite field, we define the zeta function of a noncommutative K3 surface, an invariant under Fourier–Mukai equivalence. This invariant can be used to study the geometricity of K3 categories over non-algebraically closed fields. We also give a nontrivial restriction on the possible Weil polynomials of the K3 category of a cubic fourfold.

Tristan Phillips [Postdoc] (Dartmouth College):*Counting abelian surfaces*

ABSTRACT. How many abelian varieties have a given property? In the case of elliptic curves (i.e., abelian varieties of dimension 1) there have been many recent results which count the number of elliptic curves with a given torsion subgroup, level-structure, or which satisfy a set of local conditions. In this talk I will discuss joint work with Tyler Genao, Freddy Saia, Tim Santens, and John Yin, in which we count geometric isomorphism classes of certain abelian surfaces. In the talk we will encounter Shimura curves, weighted projective stacks, and count points of bounded height on some conic bundles.

Zachary Porat [Student] (Wesleyan University):*Computations Directly on the Cuspidal Cohomology of Congruence Subgroups of* $\mathrm{SL}(3,\mathbb{Z})$

ABSTRACT. Ash, Grayson, and Green computed the action of Hecke operators on a certain subspace of the cohomology of low-level congruence subgroups of $\mathrm{SL}(3,\mathbb{Z})$. This subspace contains the cuspidal cohomology, which is of primary interest. For prime level less than 100, they found four levels at which nonzero cuspidal classes arose and determined local factors for the associated L-functions. In this talk, we extend their work, introducing a method that allows for computing the action of Hecke operators directly on the cuspidal cohomology. Using this method, we obtain data for prime level less than 1500, finding four additional levels at which nonzero cuspidal classes appear and calculating local factors for two of these levels.

Frédéric Rochon [Faculty] (UQAM) :*Growth of torsion in the cohomology of some arithmetic groups of* $\mathbb{Q}$*-rank one*

ABSTRACT. Given a number field F with ring of integers $\mathcal{O}_F$, one can associate to any torsion free subgroup of $SL(2,\mathcal{O}_F)$ of finite index a complete Riemannian manifold of finite volume with fibered cusp ends. For natural choices of flat vector bundles on such a manifold, we will show that analytic torsion is identified with the Reidemeister torsion of the Borel-Serre compactification. Following an approach introduced by Bergeron and Venkatesh, we will then explain how this can be used to obtain exponential growth of torsion in the cohomology for sequences of congruence subgroups. This is a joint work with Werner Müller.

Martí Roset Julià [Student] (McGill University) :*p-adic rigid cocycles for* SL_n *and explicit class field theory for totally real fields*

ABSTRACT. We outline an approach to generalize the construction of the Dedekind–Rademacher rigid analytic cocycle of Darmon, Pozzi, and Vonk to the case of SL_n. This construction departs from the Eisenstein class of a torus bundle, analyzed through the framework developed by Bergeron, Charollois, and García. We conclude with speculations on the relationship between these cocycles and Gross–Stark units. This is ongoing joint work with Peter Xu.

Cihan Sabuncu [Student] (Université de Montréal):*Visiting early at prime times*

ABSTRACT. Analytic Number Theory has witnessed many breakthroughs on fundamental problems in recent years, including Zhang, Maynard and Tao's celebrated results on bounded prime gaps. In this talk, I will explain that given $m > 1$ and a sufficiently large q, how to adapt their method to establish the existence of an arithmetic

progression with common difference q for which the m-th least prime in such progression is $\ll_m q$, a result that can be seen to be best possible. As we vary over progressions instead of fixing a particular one, the nature of our problem differs from others in the literature. Furthermore, inspired by the simple yet far-reaching Poincaré recurrence theorem, I will discuss a generalization to measure-preserving systems. This is joint work with Tony Haddad and Sun-Kai Leung.

Jeremy Schlitt [Student] (Université de Montréal) :*The Multiplication Table Problem for Integers with Restricted Prime Factors*

Abstract. A famous problem of Erdős asks for an estimate on the number of distinct integers appearing in the N-by-N multiplication table, as $N \to \infty$. In this talk, we examine multiplication tables where the row and column numbers are chosen from a specially constructed set of integers, so that all prime factors appearing in the table come from a set of primes with asymptotic density δ. As δ varies, we exhibit a change of behavior around a critical "magic delta" point - a feature not present in the $\delta = 1$ case of Erdős.

Jakob Streipel [Postdoc] (University at Buffalo) :*Stechkin's trick for improving zero-free regions*

Abstract. In this talk we'll discuss a somewhat forgotten inequality from the 1970's due to S. B. Stechkin, and how it can be used to improve zero-free regions of L-functions by combining it with the standard approach due to de la Vallée Poussin in 1896. This simple inequality lets one improve any zero-free region argument that uses a so-called explicit formula, and as an example we will talk about recent and ongoing joint work with Steven Creech, Alia Hamieh, Simran Khunger, Kaneenika Sinha, and Kin Ming Tsang where we use this trick (and other tools) to find a quite good explicit zero-free region for L-functions of modular forms.

Akshay Venkatesh [Faculty] (IAS) :*Quadratic structures and arithmetic statistics*

Abstract. Cohen and Lenstra studied the statistical behavior of class groups, and found striking regularity reflecting the underlying algebraic structure. I will explore some situations where the class groups behave anomalously, and discuss a new algebraic structure that seems to explain this. Joint work with Artane Siad (IAS).

William Verreault [Student] (University of Toronto) :*Moments of random multiplicative functions over functions fields*

Abstract. Little is known about the distribution of the partial sums of random multiplicative functions defined over integers, but the order of magnitude of all moments has been recently determined by Harper. Building on recent work extending multiplicative and probabilistic number theory to the function field setting, we study the even natural moments of partial sums of Steinhaus and Rademacher random multiplicative functions defined over function fields. Using analytic arguments that parallel previous work over the integers as well as new combinatorial arguments special to the function field setting, we obtain an exact expression for the fourth moment and an asymptotic expression for the higher natural moments in the limit as $qN \to \infty$.

Max Weinreich [Postdoc] (Harvard University):*Arithmetic dynamics of billiards*

Abstract. Billiards is a dynamical system that models a point particle bouncing off some walls. If these walls are algebraic plane curves over a number field, we can speak of the Weil height of the billiard ball whenever it meets the wall. How does this height grow over time for a typical starting position of the ball? This growth rate, the arithmetic degree, conjecturally agrees with the dynamical degree, a purely algebraic invariant that we compute in this talk.

Jiachang Xu [Postdoc] (Bulgarian Academy of Sciences/University of Miami) :*On the structure of the complement of skeleton*

Abstract. Berkovich spaces give a formalism for constructing spaces of valuations on varieties over nonarchimedean fields. As such they encode a great deal of information from birational geometry. The most notable invariant is the essential skeleton, a subset of the Berkovich space corresponding to the valuations monomial on strata of a dlt minimal model of X. Inspired by Mori's conjecture in birational geometry, we conjecture that the

essential skeleton is the complement of the images of all open units disks, which are analytic objects analogous to families of rational curves. This is a joint work with Morgan Brown and Muyuan Zhang.

Benjamin York [Student] (University of Connecticut): *Models of CM elliptic curves with prescribed ℓ-adic Galois image*

Abstract. In 2015, Zywina provided Weierstrass models of elliptic curves with complex multiplication (CM) over $\mathbb{Q}$ with a prescribed image of its mod-ℓ Galois representation. In 2018, Lozano-Robledo provided a classification for ℓ-adic Galois representations attached to elliptic curves with complex multiplication (CM). In this talk, we will discuss a classification of Weierstrass models for CM elliptic curves with specified ℓ-adic Galois representation, and discuss our methods for proving this classification. This is joint work with Enrique González-Jiménez and Álvaro Lozano-Robledo.