Conférence de
THÉORIE DES NOMBRES QUÉBEC-MAINE

Université Laval, Québec
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en l'honneur d'Andrew Granville à l'occasion de son 60ième anniversaire

SOUTIENS FINANCIERS

TIMC (Tutte Institute for Mathematics and Computing)

CICMA (Centre Interuniversitaire en Calcul Mathématique Algébrique)
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ORGANISATEURS

Hugo Chapdelaine
Claude Levesque

Exposés /Talks

Below are to be found the titles and abstracts of the talks of the (preliminary) program. They are listed in alphabetic order according to the first letter of the last name of the speaker which has been submitted on the google sheet registration form.

Résumés / Abstracts

Patrick Allen [**Faculty**] (McGill U.) *Minimal $R = T$ in the absence of minimal lifts*

Abstract. Wiles's famous theorem that all semistable elliptic curves over the rationals are modular follows from $R = T$ theorems, which identify certain parameter rings for Galois representations with Hecke algebras. These $R = T$ theorems are first proved in the so-called minimal case, by Taylor and Wiles, and this is used as an input for the general case. Necessary for the minimal case is the existence of minimal lifts of mod p modular forms, which follows from work of Carayol and Ribet, except for some particular cases that are excluded by the technical Taylor-Wiles hypothesis. We'll consider one of these excluded case and what one can say about minimal $R = T$ theorems for this example, attempting to explain a link between some derived structure on the Galois side with the orbifold structure on the modular side. This is joint work in progress with Preston Wake.

Louis-Pierre Arguin [**Faculty**] (CUNY) *Large values of the Riemann zeta function on the critical line*

Abstract. The interplay between probability theory and number theory has a rich history of producing deep results and conjectures. Important instances include the works of Erdös, Kac, Selberg, Montgomery, Tenenbaum, Soundararajan and Granville, to name a few. This talk will review recent results in this spirit where the insights of probability have led to a better understanding of large values of the Riemann zeta function on the critical line. In particular, we will highlight a new connection between the values of the zeta functions in short intervals and a stochastic process known as branching random walk. This is based on joint works with Emma Bailey, Paul Bourgade and Maksym Radziwill.

Eran Assaf [**Postdoc**] (Dartmouth College) *Plane models of modular curves*

Abstract. We prove that there are finitely many modular curves that admit a smooth plane model. Moreover, if the degree of the model is greater than or equal to 19, no such curve exists. For modular curves of Shimura type we show that none can admit a smooth plane model of degree $5, 6$ or 7. Further, if a modular curve of Shimura type admits a smooth plane model of degree 8 we show that it must be a twist of one of four curves.

David Ayotte [**Student**] (Concordia U.) *Drinfeld Modular Forms and Their Values at CM Points*

Abstract. The goal of this talk is to present an ongoing work on a function field analogue of a theorem by Shimura. More precisely, we show that the special value at a CM point of an arithmetic Drinfeld modular form of arbitrary rank lies in the Hilbert class field of the CM field up to a period, independent of the chosen modular form.

Petar Bakic [**Postdoc**] (U. of Utah) *Exceptional Howe correspondences and Arthur packets for G_2*

Abstract. The theory of local theta correspondence is built up from two ingredients: a reductive dual pair inside a symplectic group, and a Weil representation of its metaplectic cover. Exceptional correspondences arise similarly: dual pairs inside exceptional groups can be constructed using so-called Freudenthal Jordan algebras, while the minimal representation provides a suitable replacement for the Weil representation. Focusing on a particular dual pair, we explain how one obtains Howe duality for the correspondence in question. Finally, we discuss how these results can be used to construct certain families of Arthur packets for the exceptional group G_2.

Carlos Caralps Rueda [**Student**] (Universitat Auntònoma de Barcelona and Concordia U.) *Computation of values of zeta functions using Eisenstein Series*

Abstract. In this talk we shall explain how Eisenstein Series can be used to compute values of zeta functions using an idea of Colmez. We will start by presenting the computational method, and its related concepts, in the simplest setting namely when the base field is $\mathbb{Q}$ and the corresponding zeta function is the classical Riemann zeta function. Then we shall generalize the procedure to zeta functions of real quadratic fields. In particular, when applied to the value at $s = 1$ of a special class of zeta functions, this provides a way for computing Stark's units over real quadratic fields with the help of the LLL algorithm.

Samit Dasgupta [**Faculty**] (Duke U.) *Stark's Conjectures and Hilbert's 12th Problem*

Abstract. In this talk we will discuss two central problems in algebraic number theory and their interconnections: explicit class field theory (also known as Hilbert's 12th Problem), and the special values of L-functions. The goal of explicit class field theory is to describe the abelian extensions of a ground number field via analytic means intrinsic to the ground field. Meanwhile, there is an abundance of conjectures on the special values of L-functions at certain integer points. Of these, Stark's Conjecture has special relevance toward explicit class field theory. This conjecture states the existence of certain canonical elements in CM abelian extensions of totally real fields. Next I will state a conjectural exact formula for these Brumer-Stark units that has been developed over the last 15 years. I will conclude with a description of my work with Mahesh Kakde that proves these conjectures away from $p = 2$, thereby giving an explicit class field theory for totally real fields.

Chantal David [**Faculty**] (Concordia U.) *On the vanishing of twisted L-functions of elliptic curves over function fields*

Abstract. Let E be an elliptic curve over $\mathbb{Q}$, and let χ be a Dirichlet character of order ℓ for some prime $\ell \geq 3$. Heuristics based on the distribution of modular symbols and random matrix theory have led to conjectures predicting that the vanishing of the twisted L-functions $L(E, \chi, s)$ at $s = 1$ is a very rare event (David-Fearnley-Kisilevsky and Mazur-Rubin). In particular, it is conjectured that there are only finitely many characters of order $\ell > 5$ such that $L(E, \chi, 1) = 0$ for a fixed curve E.

We investigate in this talk the case of elliptic curves over function fields. For Dirichlet L-functions over function fields, Li and Donepudi-Li have shown how to use the geometry to produce infinitely many characters of order $\ell \geq 2$ such that the Dirichlet L-function $L(\chi, s)$ vanishes at $s = 1/2$, contradicting (the function field analogue of) Chowla's conjecture. We show that their work can be generalized to constant curves $E/\mathbb{F}_q(t)$, and we show that if there is one Dirichlet character χ of order ℓ such that $L(E, \chi, 1) = 0$, then there are infinitely many, leading to some specific examples contradicting (the function field analogue of) the number field conjectures on the vanishing of twisted L-functions. Such a dichotomy does not seem to exists for general curves over $\mathbb{F}_q(t)$, and we produce empirical evidence which suggests that the conjectures over number fields also hold over function fields for non-constant $E/\mathbb{F}_q(t)$.

This is joint work with A. Comeau-Lapointe, M. Lalin and W. Li.

Julie Desjardins [**Faculty CLTA**] (U. of Toronto) *Torsion points and concurrent lines on Del Pezzo surfaces of degree* 1

Abstract. The blow up of the anticanonical base point on X, a del Pezzo surface of degree 1, gives rise to a rational elliptic surface E with only irreducible fibers. The sections of minimal height of E are in correspondence with the 240 exceptional curves on X. A natural question arises when studying the configuration of those curves: If a point of X is contained in "many" exceptional curves, it is torsion on its fiber on E ? In 2005, Kuwata proved for del Pezzo surfaces of degree 2 (where there is 56 exceptional curves) that if "many"equals 4 or more, then yes. With Rosa Winter, we prove that for del Pezzo surfaces of degree 1, if "many" equals 9 or more, then yes, but we find counterexamples where a torsion point lies at the intersection lies at the intersection of 7 exceptional curves.

Cédric Dion [**Student**] (U. Laval) *Functional equation for supersingular abelian varieties*

Abstract. Fix an odd prime number p. Let K be an imaginary quadratic field where p splits. Let A be an abelian variety defined over K with good supersingular reduction at both primes above p. In this talk, we investigate some aspect of the Iwasawa theory for A over the $\mathbb{Z}_p^2$-extension of K. We begin by giving an overview of the relevant Selmer groups building on the work of Büyükboduk–Lei and Loeffler–Zerbes. Using the theory of Γ-systems, we then prove an algebraic functional equation involving the dual of our Selmer groups.

Anthony Doyon [Student] (U. Laval) *Lambda-invariants of Mazur-Tate elements attached to Ramanujan's tau function*

ABSTRACT. Let Δ be the unique normalized cuspidal modular form of weight 12 and level 1 given by the following q-expansion

$$\Delta(z) = q \prod_{n\geq 1}(1-q^n)^{24} = \sum_{n\geq 1} \tau(n) q^n$$

where $\tau : \mathbb{N} \to \mathbb{Z}$ is known as Ramanujan's tau function. For $p \in \{3, 5, 7\}$, we show that Δ is congruent to an infinite family of Eisenstein series $E_{k,\chi,\psi}$ modulo p for explicit choices of k and Dirichlet characters χ and ψ. This congruence suggests that there exist strong p-adic links between the modulars symbols attached to Δ and to $E_{k,\chi,\psi}$. We exhibit such links by proving a formula describing the Iwasawa λ-invariants of the p-adic Mazur–Tate elements attached to Δ. This is joint work with Antonio Lei.

Xavier Génereux [Student] (UdeM) *La propriété de Northcott de fonctions zeta sur des familles d'extensions algébriques*

ABSTRACT. En 2020, Pazuki et Pengo ont défini une propriété de Northcott pour les valeurs spéciales de fonctions zeta associées à des corps de nombres et certaines fonctions motiviques L. Je présenterai l'étude de la question analogue sur les corps de fonctions, et ce, pour n'importe quelle évaluation complexe. Finalement, je retournerai à la question sur les corps de nombres et présenterai des résultats récents qui considèrent le plan complexe au complet.

Mathilde Gerbelli-Gauthier [Postdoc] (CRM/McGill) *Towards a converse theorem for* mod ℓ *gamma factors*

ABSTRACT. How can one describe all irreducible representations of a finite group? One usually first learns about the character of a representation, which attaches a number to each conjugacy class. Can we tell apart representations using even fewer invariants? For the groups GL_2 over a finite field, the local converse theorem of Piatetski-Shapiro says that γ factors, whose construction are motivated by the local Langlands program, fully determine irreducible representation. But what if the representations are valued in a field of positive characteristic? For fields of characteristic coprime to p, I will report on the development of a mod ℓ Gamma factor. After showing that the naive reduction modulo ℓ of the γ factor is not always complete invariant, I will construct a candidate for a replacement, and describe some ongoing extensions to GL_n. This is joint work in progress with Jacksyn Bakeberg, Heidi Goodson, Ashwin Iyengar, Gil Moss, and Robin Zhang.

Andrew Granville [Faculty] (U. de Montréal) *K-rational points on curves*

ABSTRACT. Mazur and Rubin's "Diophantine stability" program suggests asking, for a given curve C, over what fields K does C have rational points, or at least to study the degrees of such K. We study this question for planar curves C from various perspectives and relate solvability to the shape of C's Newton polygon (the real original one that Newton worked with, not a p-adic one which are frequently used in arithmetic geometry research). This is joint work with Lea Beneish

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Nathan Grieve [Faculty] (RMC/Carleton/UQAM) *On accumulation and complexity of rational points in projective varieties*

ABSTRACT. I will explain the manner in which the geometry of numbers arises when studying Diophantine arithmetic questions for rational points in projective varieties. Specifically, I will survey some of the recent developments that surround Schmidt's Subspace Theorem (in its various formulations) and the height inequalities of P. Vojta. This progress has been made possible by key insights of Corvaja-Zannier, P. Autissier, A. Levin, Evertse-Ferretti, Ru-Vojta, C. Li, K. Fujita and many others. There is a strong interplay with higher dimensional birational and toric geometry (including K-stability). Finally, I intend to say a few words about the question of potential Zariski density of integral points.

Asimina Hamakiotes [Student] (U. of Connecticut) *Computing the proportion of sneaky primes for pairs of elliptic curves*

ABSTRACT. Let ℓ be a prime number and let E and E' be ℓ-isogenous elliptic curves defined over a finite field k of characteristic $p \neq \ell$. Suppose the groups $E(k)$ and $E'(k)$ are isomorphic, but $E(K) \not\simeq E'(K)$, where K is an ℓ-power extension of k. We have previously shown that, under mild rationality hypotheses, the case of interest is when $\ell = 2$ and K is the unique quadratic extension of k. In this talk we determine the likelihood of such an occurrence by fixing a pair of 2-isogenous elliptic curves E, E' over $\mathbf{Q}$ and asking for what proportion of primes p do we have $E(\mathbf{F}_p) \simeq E'(\mathbf{F}_p)$ and $E(\mathbf{F}_{p^2}) \not\simeq E'(\mathbf{F}_{p^2})$.

Sun Kai Leung [Student] (U. de Montréal) *Dirichlet law for factorization of integers, polynomials and permutations*

ABSTRACT. Let $k \geq 2$ be an integer. We prove that factorization of integers into k parts follows the Dirichlet distribution $\mathrm{Dir}\left(\frac{1}{k}, \ldots, \frac{1}{k}\right)$ by multidimensional contour integration, thereby generalizing the Deshouillers-Dress-Tenenbaum (DDT) arcsine law on divisors where $k = 2$. The same holds for factorization of polynomials or permutations. Dirichlet distribution with arbitrary parameters can be modelled similarly.

Erik Holmes [Postdoc] (U. of Calgary) *A Study of Shapes*

ABSTRACT. The shape of a lattice is defined to be its equivalence class up to scaling, rotation, and reflection. For a number field, K, there are a few natural lattices that we can ask about the 'shape' of: our primary focus for this talk will be on the shape of a lattice coming from Minkowski's embedding $j : \mathcal{O}_K \to K_{\mathbb{R}}$. Shapes are typically studied as we vary over particular families of fields: most commonly as degree n fields and often with additional Galois restrictions. We highlight a few of the families for which these shapes have been studied and provide an overview of our current projects in this area. Specifically we will discuss our recent work on the shape of pure, prime degree, number fields including the (regularized) equidistribution of these shapes. We will also mention joint work with Rob Harron in which we define a natural refinement of shapes to study the case of non-Galois sextic fields (i.e. those with absolute Galois group $C_3 \wr C_2$): here we prove equidistribution results and observe a relationship between the study of shapes and the log terms in Malle's conjecture. Time permitting we will discuss ongoing work regarding the shape of another natural lattice associated to number fields, the unit-log lattice.

Andrew Keisling, Xuyan Liu, Annika Mauro, Zoe McDonald, Santiago Miguel Velazquez Iannuzzelli, Jack Miller [Students], Steven Miller [Faculty] *An Excised Orthogonal Model for Families of Cusp Forms*

ABSTRACT. According to the Katz-Sarnak philosophy, the zeros of certain families of L-functions are distributed like the eigenvalues of special families of random matrices as the conductor goes to infinity. For finite conductor, in families of L-functions coming from twists of Hecke eigenforms, the central values are discretized according to the Kohnen-Zagier formula, leading to a repulsion of zeros from the central point. To capture this repulsion in the zero-statistics using random matrix theory in the weight 2 case, Dueñez, Huynh, Keating, Miller, and Snaith developed an *excised orthogonal model* in which they filter the random matrices using a cutoff value reflecting the discretization. We discuss our work extending this model to higher weights. Using the cutoff value predicted by Kohnen-Zagier, we provide computational evidence for an excised orthogonal model for families of modular forms of weights 2, 4, and 12. We test the statistical fitness of our model by observing if the excision parameter given by small-twist data accurately predicts the distribution of zeros of larger twists. The stability of the excision parameter is suggestive of an underlying analytic proportionality constant.

Paul Kinlaw [Faculty] (Dickinson College) *Higher Mertens constants for almost primes II*

ABSTRACT. A k-almost prime is a product of k primes, counted with multiplicity. In last year's talk, we covered a generalization of Mertens' second theorem, giving an asymptotic estimate for the partial sums of reciprocals of k-almost primes up to x. This is a polynomial of degree k in $\log\log x$, plus an error term which tends to zero. Also, for the case $k = 2$, we replaced the error term with a high-precision asymptotic expansion with terms $\alpha_j/\log^j x$, $j = 1, \ldots, N$, and error $O_N(1/\log^{N+1} x)$ for any N. In this talk, we cover a proof of an asymptotic formula for α_j, specifically $2^{j-1}(j-1)!/j$, which had been conjectural. We also give a sharper estimate conditional on RH. Furthermore, we extend the high-precision expansion to arbitrary k. The higher Mertens constants which arise have an unexpected connection with the proof of the asymptotic formula for α_j. This is joint work with Jonathan Bayless (UMaine Augusta) and Jared Lichtman (Oxford U.).

Jean-Marie De Koninck [Retired] (Université Laval) *Consecutive integers divisible by a power of their largest prime factor*

ABSTRACT. Given an integer $n \geq 2$, let $P(n)$ stand for its largest prime factor. Given integers $k \geq 2$ and $\ell \geq 2$, consider the set $E_{k,\ell}$ of those integers $n \geq 2$ for which $P(n+i)^\ell \mid n+i$ for $i = 0, 1, \ldots, k$. Each of these sets is very thin. For instance, the smallest element of $E_{3,2}$ is 1 294 298, the smallest known element of $E_{3,3}$ has 77 digits and no elements of $E_{4,2}$ are known, even though all these sets are believed to be infinite. In this talk, using elementary, analytic and probabilistic approaches, we will shed some light on these sets and raise several open problems. This is joint work with Nicolas Doyon, Florian Luca and Matthieu Moineau.

Harun KIR [**Student**] (Queen's U.) *Imprimitive refined Humbert invariant*

Abstract. In this talk, I will give a classification of imprimitive ternary forms which are equivalent to some refined Humbert invariant.

Matilde Lalin [**Faculty**] (Université de Montréal) *Aspects of Dynamical Mahler Measure*

Abstract. The Mahler measure of a multivariable polynomial or rational function P is given by the integral of $\log|P|$ where each of the variables moves on the unit circle and with respect to the Haar measure. We consider a dynamical generalization of Mahler measure and present dynamical analogues of various results from the classical Mahler measure as well as examples of formulas allowing the computation of the dynamical Mahler measure in certain cases.

Joint work with Annie Carter, Michelle Manes, Alison Beth Miller, and Lucia Mocz

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Youness Lamzouri [**Faculty**] (U. de Lorraine / CRM-CNRS) *The distribution of quadratic character sums*

Abstract. In this talk, I will present recent results on the distribution of the maximum of quadratic character sums, as well as some applications. In particular, our work improves results of Montgomery and Vaughan, and gives strong evidence that the Omega result of Bateman and Chowla for quadratic character sums is optimal. Our results are motivated by a recent work of Bober, Goldmakher, Granville and Koukoulopoulos, who proved similar results for the family of all non-principal characters modulo a large prime, but our approach is different and relies principally on the quadratic large sieve. We shall also describe two applications of our results. The first concerns the positivity of sums of the Legendre symbol, a question that was considered by Montgomery. The second, joint with Ayesha Hussain, investigates the distribution of character paths formed with the Legendre symbol.

Adam Logan [**researcher**] (Government of Canada and Carleton U.) *Modular Calabi-Yau threefolds*

Abstract. Every elliptic curve over $\mathbb{Q}$ is modular: in other words, the number of points mod p can be expressed in terms of the Fourier coefficients of an eigenform for the Hecke operators. Conversely, every such eigenform corresponds to an elliptic curve. The first statement has been generalized to higher dimensions, but the second appears to be more difficult. Mazur, van Straten, and others have asked whether every eigenform of weight 4 is realized by a "rigid Calabi-Yau threefold". We will explain the terms in this abstract and describe a construction that produces at least 15 candidate realizations for modular forms for which previously none was available.

Jonathan Love [**Postdoc**] (McGill U.) *Rational equivalences on surfaces using hyperelliptic subcurves*

Abstract. A conjecture of Bloch and Beilinson predicts that for a smooth projective variety X over an algebraic number field, the kernel of the Abel-Jacobi map of X is a torsion group. If X is a curve then this kernel is trivial, but in higher dimensions there is hardly any evidence for this conjecture. In this talk we will describe a technique that can be used to construct infinitely many nontrivial examples of surfaces that satisfy a weaker form of the Bloch-Beilinson conjecture. This is joint work with Evangelia Gazaki.

David Lowry-Duda [**Postdoc**] (ICERM) *Counting Number Fields of Bounded Discriminant*

Abstract. In this talk, we discuss recent improvements on counts of number fields up to a given discriminant for number fields of small degree. These improvements use a combination of algebraic ingredients and Fourier analysis.

Caleb McWhorter [**Faculty**] (St. Thomas Aquinas College) *Torsion Subgroups of Rational Elliptic Curves over Odd Degree Galois Fields*

Abstract. The last 30 years has been a deep well-spring of progress in the classification of the possibilities for torsion subgroups of (rational) elliptic curves over number fields. Concretely, let $\Phi_{\mathbb{Q}}(n)$ denote the set of isomorphism classes of torsion subgroups of rational elliptic curves E over a number field K as E varies over all rational elliptic curves and K varies over all number fields of degree n. This talk will give a brief overview of the n for which results are known as well as related questions. The remainder of the talk will focus on the classification when n is odd and one restricts to Galois fields, along with a number of open questions.

Jack Miller and Annika Mauro [**Students**] (Yale U., Stanford U.) *Extending the support of* 1- *and* 2-*level densities for cusp form* L-*functions under square-root cancellation hypotheses.*

Abstract. Following the Katz–Sarnak philosophy, we investigate the low-lying zeroes of L-functions associated to weight k cusp forms by considering their n-level densities We extend the work of Iwaniec–Luo–Sarnak, specifically the calculations that use a hypothesis on the growth of complex exponential sums over primes. Under the assumption known as *Hypothesis* S, Iwaniac–Luo–Sarnak are able to increase the Fourier support of an even test function $\phi \in C_0^\infty$ from the open interval $(-2, 2)$ to $(-22/9, 22/9)$. We notice that the arguments in Iwaniec–Luo–Sarnak's manuscript lead to an increase in support of the 1-level density which is greater than what they had originally claimed, up to $(-5/2, 5/2)$.

We formulate a natural analog of Hypothesis S which we call *Hypothesis* T that extends the idea of square-root cancellation to the 2-level setting. We are able to increase the Fourier support of a product of even test functions $\phi(x, y) = \phi_1(x)\phi_2(y)$ from the open diamond $|x| + |y| < 2$ to $|x| + |y| < \frac{12}{5}$. This is a natural extension of the work of Iwaniec–Luo–Sarnak, and provides further evidence for the Katz–Sarnak density conjecture relating analytic number theory and random matrix theory under this hypothesis.

Katharina Müller [**Postdoc**] (Université Laval) *Iwasawa theory for* $\mathbb{Z}_p^\ell$ *covers of graphs*

Abstract. We will describe how to generalize work of Gonet for $\mathbb{Z}_p$-covers of finite graphs to $\mathbb{Z}_p^l$-covers. If time allows we will also discuss a "main conjecture" in this setting.

Carlo Pagano [**Faculty**] (Concordia U.) *On the negative Pell equation and applications*

Abstract. I will present recent developments on the Cohen–Lenstra heuristics on the class groups of quadratic fields, explain how they lead to a resolution of Stevenhagen's conjecture on the asymptotic number of real quadratic fields having an integral unit of negative norm and overview a few problems that can now be approached by similar techniques. This is joint work with Peter Koymans.

Marti Roset Julia [**Student**] (McGill U.) *The Gross–Kohnen–Zagier formula via p-adic uniformization*

Abstract. Let S be a set of rational primes of odd cardinality containing infinity and a rational prime p. We can associate to S a Shimura curve X defined over $\mathbb{Q}$. The Gross–Kohnen–Zagier formula states that certain generating series of Heegner points of X are modular forms of weight 3/2 valued in the Jacobian of X. We will state this formula and outline a new approach to prove it using the theory of p-adic uniformization. This is joint work in progress with Lea Beneish, Henri Darmon and Lennart Gehrmann.

Giovanni Rosso [**Faculty**] (Concordia U.) *Hirzebruch–Zagier cycles in p-adic families and adjoint L-values*

Abstract. Let E/F be a quadratic extension of totally real fields. The embedding of the Hilbert modular variety of F inside the Hilbert modular variety of E defines a cycle, called Hirzebruch–Zagier cycle. Thanks to work of Hida and Getz–Goreski, it is known that the integral of a Hilbert modular form g for E over this cycle detects if g is the base change of a Hilbert modular form for f, and in this case the value of the integral is related to the adjoint L-function of f. In this talk we shall present joint work with Antonio Cauchi and Marc-Hubert Nicole, where we show that the Hirzebruch–Zagier cycles vary in families when one considers deeper and deeper levels at p. We shall present applications to Λ-adic Eisenstein series and adjoint p-adic L-functions.

Praneel Samanta [**Student**] (U. of Iowa) *Double Square Moments and Bounds for Resonance Sums of Cusp Forms*

Abstract. Let f and g be holomorphic cusp forms for the modular group $SL_2(\mathbb{Z})$ of weight k_1 and k_2 with Fourier coefficients $\lambda_f(n)$ and $\lambda_g(n)$, respectively. For real $\alpha \neq 0$ and $0 < \beta \leq 1$, consider a smooth resonance sum $S_X(f, g; \alpha, \beta)$ of $\lambda_f(n)\lambda_g(n)$ against $e(\alpha n^\beta)$ over $X \leq n \leq 2X$. Double square moments of $S_X(f, g; \alpha, \beta)$ over both f and g are nontrivially bounded when their weights k_1 and k_2 tend to infinity together. By allowing both f and g to move, these double moments are indeed square moments associated with automorphic forms for $GL(4)$. These bounds reveal insights into the size and oscillation of the resonance sums and their potential resonance for $GL(4)$ forms when k_1 and k_2 are large.

Siva Sankar Nair [student] (U. de Montréal) *An invariant property of Mahler measures*

Abstract. The Mahler measure of a polynomial $P(x_1, x_2, \ldots, x_n)$ is the average value of $\log |P|$ along the unit torus $\mathbb{T}^n$ defined by $|x_i| = 1$ for all i. If P is univariate, this measure is given by Jensen's formula in terms of its roots, and in the multivariable case, it has been observed that it evaluates to special values of L-functions. Oftentimes, a numerical experiment leads to a conjecture equating the Mahler measures of certain polynomials to these special values. In this talk, we shall investigate an interesting invariant property that provides a method to extend identities involving Mahler measures and also resolve some conjectures along the way.

Ruiran Sun [Postdoc] (McGill U.) *One-pointed Shafarevich's conjecture for moduli spaces of canonically polarized manifolds*

Abstract. Motivated by Shafarevich's conjecture, Arakelov-Parshin proved the following finiteness result: for every curve C, the set of isomorphism classes of nonconstant morphisms $C \to M_g$ is finite ($g \geq 2$). For moduli stacks parametrizing higher dimensional varieties Arakelov-Parshin's finiteness theorem fails for trivial reason, i.e. the existence of product families. In this talk we will explain that this is somehow the only obstruction: the finiteness theorem holds true for the Hom set of "pointed" curves (in which the product families are excluded). We also discuss some application of this result. This is a joint work with Ariyan Javanpeykar, Steven Lu and Kang Zuo.

William Verreault [Student] (U. Laval) *Asymptotics for compositions under constraints*

Abstract. Compositions and partitions that respect a given system of linear Diophantine inequalities have been studied, for instance, by Andrews et al. in the context of MacMahon's partition analysis, by Corteel et al. in a series of papers on compositions defined by inequalities, and are related more generally to systems of linear Diophantine equations as studied by Stanley. While it is very hard to say anything about these compositions in general, we show that we can obtain asymptotics for the number of such compositions by adopting a probabilistic viewpoint and making connections with broken stick problems.

Ciaran Schembri [Postdoc] (Dartmouth College) *Reducing models for branched covers of the projective line*

Abstract. We discuss a method for reducing models of curves equipped with a map to the projective line which is unramified away from three points. Using the ramified points of the map, we can compute "small" functions supported at these points to produce "small" plane models of the original curve. It is then possible to rescale a plane model in an optimal way to reduce the size of the coefficients, for which we use an integer linear program. We implemented and ran the algorithm on a database of Belyi maps in the LMFDB with often very favourable results. This is joint work with Sam Schiavone and John Voight.

Andrew Schultz [Faculty] (Wellesley College) *Detecting summand types in the module structure of square power classes over biquadratic extensions*

Abstract. If K/F is a biquadratic extension, recent investigations have determined the module structure of $K^\times/K^{\times 2}$ as a module over $\mathrm{Gal}(K/F)$. Although the modular representation theory provides an infinite number of indecomposable types over this particular group ring, it turns out that at most 9 summand types appear in this decomposition. The multiplicities of each summand type were originally determined according to the solvability of certain module-theoretic equations, but more recent work has been able to rephrase these multiplicities in terms of quaternion algebras. In this talk we survey these results and give some specific examples.

Naomi Tanabe [Faculty] (Bowdoin College) *Moments of Rankin-Selberg L-functions*

Abstract. In this talk, we will study the asymptotic behaviors of the first and second moments of L-functions associated with two modular forms in the weight aspect on average. This is ongoing joint work with Alia Hamieh.

Gary Walsh [Faculty] (U. of Ottawa) *Lower bounds for ranks on a family using Diophantine methods*

Abstract. Using methods from Diophantine analysis, such as Baker's theorem, Runge's theorem, and the Pellian equation, we show how to construct curves of the form $y^2 = f(x) + m^2$, having rank at least 3, and often considerably larger.

Jiacheng Xia [**Postdoc**] (U. Laval) *Towards some cases of the unitary Kudla conjecture over CM fields*

Abstract. As an analogue of the classical Gross-Kohnen-Zagier theorem for higher dimensions, the unitary Kudla conjecture asserts the modularity of certain generating functions for special cycles on unitary Shimura varieties, where the special cycles are the analogues of Heegner divisors. We will report a progress towards some cases of this conjecture and its application to the Beilinson-Bloch conjecture. Following the method of Bruinier-Raum, one key ingredient is the boundary geometry of toroidal compactifications of Hermitian modular varieties, and one way to understand this is via cohomology of these modular varieties.

Peter Xu [**Student**] (McGill U.) *Higher Kato elements and equivariant complexes*

Abstract. Kato used cup products of two Siegel units to obtain remarkable arithmetic applications for modular curves. Building on recent ideas of Sharifi-Venkatesh, Kings-Sprang, and Bergeron-Charollois-Garcia, we are able to construct analogous elements for a highly general class of Shimura varieties. Except in certain "split" cases, these elements are harder to make explicit than Kato's original elements, though their archimedean regulators can be given concrete analytic formulas. However, they have a "bonus" structure of being organized into a group cocycle, which makes certain very intriguing relations between them conceptually transparent. Some ongoing and potential arithmetic applications are discussed.
