Special values of Drinfeld modular forms at CM points

David Ayotte

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Classical Theory

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Theorem (Shimura, 1975)

There exists a period $\mathbf{\Omega}_\tau \in \mathbb{C}^\times$ such that

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Ongoing work: prove the analogue for Drinfeld modular forms.

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- K_∞ the completion of K w.r.t. $|\cdot|_\infty$;
- $\mathbb{C}_\infty$ the completion of an algebraic closure of K_∞.

Analogies to have in mind

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Example. $K = \mathbb{F}_q(\mathbb{P}^1)$.

In this case, we have explicitly

$$A = \mathbb{F}_q[T], \qquad K = \mathbb{F}_q(T), \qquad K_\infty = \mathbb{F}_q((1/T)).$$

Drinfeld Period Domain

Let $r \geq 2$ be an integer.

Definition

$$\begin{aligned} \Omega^r(\mathbb{C}_\infty) &:= \mathbb{P}^{r-1}(\mathbb{C}_\infty) \setminus \{K_\infty\text{-hyperplanes}\} \\ &= \left\{ (w_1, \ldots, w_{r-1}, 1)^{\mathrm{T}} : w_i \text{ are } K_\infty\text{-linearly independant} \right\} \\ &\subset \mathbb{C}_\infty^r \end{aligned}$$

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$$\subset \mathbb{C}_\infty^r$$

Therefore, $\mathrm{GL}_r(K_\infty)$ acts on $\Omega^r(\mathbb{C}_\infty)$:

$$\gamma(w) := \Big(\underbrace{\text{last entry of } \gamma w}_{j(\gamma,w)}\Big)^{-1} \gamma w.$$

Weak Drinfeld Modular Forms

Recall. $\gamma(w) := j(\gamma, w)^{-1}\gamma w$

Let k and $r \geq 2$ be two integers.

Definition

A weak Drinfeld modular form of weight k, rank r for $\mathrm{GL}_r(A)$ is a holomorphic function $f : \Omega^r(\mathbb{C}_\infty) \to \mathbb{C}_\infty$ such that

$$f(\gamma(w)) = j(\gamma, w)^k f(w)$$

for all $\gamma \in \mathrm{GL}_r(A)$ and $w \in \Omega^r(\mathbb{C}_\infty)$.

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$$\gamma = \left(\begin{array}{c|ccc} 1 & * & \cdots & * \\ \hline 0 & & & \\ \vdots & & \mathrm{Id}_{r-1} & \\ 0 & & & \end{array}\right) \quad \Rightarrow \quad f(\gamma(w)) = f(w).$$

The u-Expansion

By rigid analysis, there exists a translation invariant parameter

$$u : \Omega^r(\mathbb{C}_\infty) \to \mathbb{C}_\infty$$

and a sequence $f_n : \Omega^{r-1} \to \mathbb{C}_\infty$ such that

$$f(w) = \sum_{n \in \mathbb{Z}} f_n(w') u_w^n$$

for $w = \binom{w_1}{w'} \in \mathcal{N} \subset \Omega^r(\mathbb{C}_\infty)$ with $w' \in \Omega^{r-1}(\mathbb{C}_\infty)$.

We call this the u-expansion of f.

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Note. One can define modular forms for arithmetic subgroup $\Gamma \leq \mathrm{GL}_r(K)$.

Example (rank 2)

- Let $A = \mathbb{F}_q[T]$. Then we have $\Omega^2(\mathbb{C}_\infty) = \mathbb{P}^1(\mathbb{C}_\infty) \setminus \mathbb{P}^1(K_\infty)$.
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We have

$$E_k^2(w) = c_0 - \sum_{\substack{a\in\mathbb{F}_q[T] \\ a \text{ monic}}} G_k(t_a(w)).$$

where G_k is a Goss polynomial and $t_a(w) = u_w^{\deg(a)} + \cdots$.

Drinfeld Modules over a Scheme

Let $j : S \to \mathrm{Spec}(A)$, $\deg(a) := \dim_{\mathbb{F}_q} A/(a)$.

Definition

A Drinfeld module of rank r over S is a pair (L, ϕ) consisting of a line bundle L over S and a ring homomorphism

$$\begin{aligned} \phi : A &\longrightarrow \mathrm{End}_S(L, +) \\ a &\longmapsto \phi_a \end{aligned}$$

such that for a trivialisation of L by open affine S-schemes we have

$$\phi_a|_{\mathrm{Spec}(B)} = \sum_{i=0}^{r \deg(a)} b_i \tau^i$$

with $b_i \in B$ such that

$$b_0 = j^*(a) \quad \text{and} \quad b_{r \deg(a)} \in B^\times.$$

Drinfeld Modules over $\mathbb{C}_\infty$

- Over $S = \mathrm{Spec}(\mathbb{C}_\infty)$, L is trivial, hence a Drinfeld module is simply determined by

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- Analytic Uniformization.

$$\left\{ \begin{array}{c} \text{Discrete projective } A\text{-modules} \\ \text{in } \mathbb{C}_\infty \text{ of rank } r \end{array} \right\} \leftrightarrow \left\{ \begin{array}{c} \text{Drinfeld modules over } \mathbb{C}_\infty \\ \text{of rank } r \end{array} \right\}$$

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- $P : \phi \to \psi$ is a morphism if $P \in \mathbb{C}_\infty\{\tau\}$ and

$$P(\tau)\phi_a(\tau) = \psi_a(\tau)P(\tau), \quad \forall a \in A.$$

Drinfeld Modular Variety

Let $N \subset A$, $r \geq 1$ and $(S, \mathcal{O}_S)$ a K-scheme.

Definition

$$F_N^r(S) := \left\{ (E, \lambda) : \begin{array}{c} E = (L, \phi) \text{ Drinfeld module over } S; \\ \lambda : (N^{-1}/A)^r \xrightarrow{\sim} \phi[N] \end{array} \right\} / \cong$$

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- Define $\boldsymbol{\omega} := \mathrm{Lie}(\mathcal{E})^\vee = \mathrm{Hom}_{\mathcal{O}_S\text{-mod}}(\mathrm{Lie}(\mathcal{E}), \mathcal{O}_S)$

Algebraic Weak Modular Forms

Definition

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Algebraic vs Analytic:

$$\mathcal{W}_k^{\mathrm{alg,r}}(N) \otimes_K \mathbb{C}_\infty \cong \bigoplus_{s \in S} \mathcal{W}_k^{\mathrm{an,r}}(\Gamma_s).$$

where S is a set of representatives for a certain double coset space such that

$$M_N^r(\mathbb{C}_\infty) \cong \bigsqcup_{s \in S} \Gamma_s \setminus \Omega^r(\mathbb{C}_\infty).$$

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Drinfeld modules with CM

Definition

A rank r Drinfeld module over $\mathbb{C}_\infty$ is said to have CM by $\mathrm{End}_{\mathbb{C}_\infty}(\phi) \otimes_A K$ if

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Drinfeld modules of rank 1 are defined over H_A =Hilbert class field of A.

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Rank reduction. A rank r Drinfeld module over $\mathbb{C}_\infty$ with CM can be seen as a rank 1 $\mathrm{End}_{\mathbb{C}_\infty}(\phi)$-Drinfeld module, $\phi : \mathrm{End}_{\mathbb{C}_\infty}(\phi) \to \mathbb{C}_\infty\{\tau\}$.

$\Rightarrow$ the field of definition of ϕ is included in $H_\phi := H_{\mathrm{End}_{\mathbb{C}_\infty}(\phi)}$.

CM points

Recall. $M_N^r(\mathbb{C}_\infty) \cong \coprod_{s\in S} \Gamma_s \setminus \Omega^r(\mathbb{C}_\infty)$.

Let $w \in \Omega^r(\mathbb{C}_\infty)$. Then w corresponds to different Drinfeld modules, one for each components $[w]_s \in \Gamma_s \setminus \Omega^r(\mathbb{C}_\infty)$.

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Fact. Suppose $r = \ell$ is prime and $w = (w_1, \ldots, w_{r-1}, 1)^{\mathrm{T}}$. Then, for every components, $\phi_{[w]_s}$ has CM by the field

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We will say that w is a CM point in $\Omega^\ell(\mathbb{C}_\infty)$.

Shimura's Result for Drinfeld Modular Forms

- Express an algebraic Drinfeld modular form as a function on triple
$$f : (E, \lambda, \omega) \longmapsto f(E, \lambda, \omega) \in L$$
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  - E is a Drinfeld module over L;
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- Evaluate it at a CM Drinfeld module and compute its pullback to $\mathbb{C}_\infty$;

- $g^*\omega = \boldsymbol{\Omega}_\phi \omega_{\mathrm{an}}$.

Katz-like description of Drinfeld modular forms

- Let $F \in \mathcal{W}_k^{\mathrm{alg},r}(N)$. Let L/K be an extension and consider a morphism $g : \mathrm{Spec}(L) \to M_N^r$.
- we have $g^*\mathcal{E} = E$.
- $\boldsymbol{\omega}$ is locally free of rank 1, therefore, if $\omega \in L$ is a basis we have
$$g^*F = f(E, \lambda, \omega)\omega^{\otimes k}, \quad f(E, \lambda, \omega) \in L$$
- if $\omega' = \mu\omega$ is another basis, we find
$$f(E, \lambda, \mu\omega) = \mu^{-k} f(E, \lambda, \omega).$$

Drinfeld Modular Forms at CM points

Let ℓ be a prime number.

Theorem (A.)

Let $f : \Omega^{\ell}(\mathbb{C}_\infty) \to \mathbb{C}_\infty$ be a weight k, rank ℓ Drinfeld modular form defined over a finite extension K_f/K. Let $w \in \Omega^{\ell}(\mathbb{C}_\infty)$ be a CM point. Then, there exists $\mathbf{\Omega}_w \in \mathbb{C}_\infty^{\times}$ such that

$$\frac{f(w)}{\mathbf{\Omega}_w^k} \in K_f H_w.$$

Thank you for listening!