Conférence de
THÉORIE DES NOMBRES QUÉBEC-MAINE

Université Laval, Québec
26 et 27 septembre 2020

SOUTIENS FINANCIERS

TIMC (Tutte Institute for Mathematics and Computing)

CICMA (Centre Interuniversitaire en Calcul Mathématique Algébrique)

ORGANISATEURS

Hugo Chapdelaine
Jean-Marie De Koninck
Antonio Lei
Claude Levesque

Exposés sur Zoom / Zoom talks

Below are to be found the titles and abstracts of the zoom talks. They are listed in alphabetic order according to the first letter of the last name of the speaker which has been submitted on the google sheet registration form.

Résumés / Abstracts

Jean-Michel Bismut (Université Paris-Sud) *Riemann-Roch and the trace formula*

Abstract. Riemann-Roch gives a local formula for Euler characteristics in algebraic geometry. Selberg's trace formula expresses the trace of a kernel as a sum of orbital integrals. I will present a "local" formula for certain semisimple orbital integrals for reductive groups in terms of the root system of the Lie algebra. Such orbital integrals will be viewed as generalized Euler characteristics, to which standard cohomological methods will be suitably adapted. The main tool of analysis is the hypoelliptic Laplacian, an object that suitably interpolates between the Laplacian and the geodesic flow.

José Ignacio Burgos Gil (ICMAT) *The hybrid topology a bridge between string theory and QFT*

Abstract. In this talk I will overview QFT and string theory and will show how the hybrid topology that relates Archimedean and non Archimedean analysis can be used as a tool to understand how string theory converges to QFT in low energies. Joint work with O. Amini, S. Bloch and J. Fresán based on an insight of P. Tourkine.

Jack Buttcane (U. Maine) *Bessel functions outside $GL(2)$*

Abstract. I will discuss the definition and construction of the Bessel-type functions occurring in the Kuznetsov trace formula in a general setting, what is currently known about them, and their applications.

Martin Cech (Concordia U.) *Mean values of real Dirichlet characters and double Dirichlet series*

Abstract. We will study the double character sum

$$\sum_{n\leq X}\sum_{m\leq Y}\left(\frac{m}{n}\right).$$

An asymptotic formula for this sum was discovered by Conrey, Farmer and Soundararajan by using Poisson summation. In this talk, we will show how to use a double Dirichlet series to obtain the same asymptotic with an improved error term.

Julie Desjardins (U. of Toronto) *Density of rational points on del Pezzo surfaces of degree* 1

Abstract. Let k be a number field and X an algebraic variety over k. We want to study the set of k-rational points $X(k)$. For example, is $X(k)$ empty ? If not, is it dense with respect to the Zariski topology? Del Pezzo surfaces are classified by their degrees d (an integer between 1 and 9). Manin and various authors proved that for all del Pezzo surfaces of degree > 1, $X(k)$ is dense provided that the surface admits at least one k-rational point (in the special case where $d = 2$ one should also require that the k-rational point lies outside a specific subset of the surface). For $d = 1$, the del Pezzo surface always has a rational point. However, we don't know it the set of rational points is Zariski-dense. In this talk, I will present a joint result with Rosa Winter in which we prove the density of rational points for a specific family of del Pezzo surfaces of degree 1 over k.

Xander Faber (IDA / Center for Computing Sciences) *Totally T-adic functions of small height*

Abstract. A nonzero algebraic number α is *totally p-adic* if its minimal polynomial (over $\mathbb{Q}$) splits completely over $\mathbb{Q}_p$. If α is not a $(p-1)st$ root of unity, then the naive logarithmic height of such an element is uniformly bounded away from zero by an equidistribution result of Bombieri/Zannier or an elementary inequality of Pottmeyer.

In this work, we introduce a geometric analogue. Fix a finite field $\mathbb{F}_q$, and consider the rational function field $\mathbb{F}_q(T)$. An algebraic function f that generates a separable extension of $\mathbb{F}_q(T)$ is *totally T-adic* if its minimal polynomial (over $\mathbb{F}_q(T)$) splits completely in the field of Laurent series $\mathbb{F}_q(\!(T)\!)$. We will discuss a lower bound for the height

of any nonconstant totally T-adic function, and we will show that functions achieving the lower bound give rise to curious algebraic curves over $\mathbb{F}_q$ with many rational points. We also investigate the limit-infimum of the heights of totally T-adic functions using a dynamical construction. This is joint work with Clay Petsche (Oregon State U.).

Paul Garret (U. of Minnesota) *Distributions, Differential Equations, and Zeros...*

Abstract. Partly joint work with E. Bombieri (IAS). Solvability of certain distributional partial differential equations $(\Delta - s(1-s))u = \theta$ on the modular curve, with $\mathrm{Re}(s) = \frac{1}{2}$, implies on-the-line vanishing of the *period* θE_s. In some interesting cases, that period is related to zeta functions and L-functions. From the other side, in some cases existence of solutions can be achieved by construction.

Lennart Gehrmann (Duisburg-Essen U./McGill U.) *Gelfand's trick for the spherical derived Hecke algebra*

Abstract. Gelfand's trick shows that the spherical Hecke algebra of a p-adic split reductive group is commutative. We generalize the method in order to show that the spherical derived Hecke algebra is graded-commutative under mild assumptions on the coefficient ring.

Jon Grantham (IDA / Center for Computing Sciences) *Extremely Pointless Curves*

Abstract. Recall that the gonality of a curve over a finite field k is the minimum degree of a k-morphism to the projective line. For a smooth projective curve of genus g over a finite field, the gonality cannot exceed $g+1$. Equality is only possible for so-called "pointless" curves, though this is not sufficient in general. We call a curve with gonality $g+1$ an "extremely pointless" curve. We determine all pairs (g, q) for which there exists an extremely pointless curve of genus g over the field with q elements, with three possible exceptions. This is joint work with Xander Faber.

Hester Graves (IDA/Center for Computing Sciences) *The abc conjecture implies that there are only finitely many Cullen numbers which are repunits*

Abstract. Assuming the abc conjecture with $\epsilon = 1$, we use elementary methods to show that for any integer $s \geq 2$, there are only finitely many s-Cullen numbers, that is numbers of the form $ns^n + 1$ that are repunits. More precisely, for fixed s, there are only finitely many positive integers n, b, and q with $n, b \geq 2$ and $q \geq 3$ such that

$$C_{s,n} = ns^n + 1 = \frac{b^q - 1}{b - 1}.$$

This is joint work with Jon Grantham.

Thomas Hulse (Boston College) *The Non-vanishing Spectrum Of Arithmetic Progressions of Squares*

Abstract. We study the asymptotics of arithmetic progressions of squares (i.e. $1, 25, 49$) by means of a triple shifted convolution sum of Jacobi theta functions. We do this by considering the sum's spectral expansion, and unexpectedly the spectrum is comprised entirely of dihedral Maass forms and so our problem can be considered as a question about L-functions of Hecke characters. Joint work with Chan Ieong Kuan, David Lowry-Duda, and Alexander Walker.

Kazim Ilhan Ikeda (Bogazici U.) *On the automorphic Langlands group*

Abstract. The aim of this talk is to introduce and discuss the properties of an unconditional topological group, whose construction depends only on a global field K, and which is related to the hypothetical automorphic Langlands group L_K of K.

Elena Kim (Pomona College) and Fernando Trejos Suarez (Yale U.) *Tinkering with Lattices: A New Take on the Erdős Distance Problem*

Abstract. The Erdős distance problem concerns the least number of distinct distances that can be determined by N points in the plane. The integer lattice with N points is known as *near-optimal*, as it spans around $O(N/\sqrt{\log(N)})$ distinct distances which is the lower bound for a set of N points (Erdős, 1946). The only previous non-asymptotic work relating to the Erdős distance problem that has been done was carried out for $N \leq 13$. We take a new non-asymptotic approach to this problem, studying the distance distribution, or in other words, the plot of frequencies of each distance of the $N \times N$ integer lattice. In order to fully characterize this distribution and determine its most common and least common distances, we adapt previous number-theoretic results from Fermat and Erdős, in order

to relate the frequency of a given distance on the lattice to the sum-of-squares formula, which determines the number of ways in which a positive integer may be written as the sum of two squares.

In order to apply our work on the lattice to the distance problem, we study the distance distributions of all its possible subsets; although this is a restricted case, we find that the structure of the integer lattice allows for the existence of subsets which can be chosen so that their distance distributions have certain properties, such as emulating the distribution of randomly distributed sets of points for certain small subsets, or that of the larger lattice itself for certain geometric configurations. We define an error which compares the distance distribution of a subset with that of the full lattice. The structure of the integer lattice allows us to take subsets with certain geometric properties in order to maximize error, by exploiting the potential for sub-structure in the integer lattice. We show these geometric constructions explicitly; further, we calculate explicit upper bounds for error for when the number of points in the subset is 4, 5, 9 or $\lceil N^2/2 \rceil$ and prove a lower bound for more general numbers of points.

Paul Kinlaw (Husson U.) *Progress towards a generalization of Mertens' theorems for almost primes*

Abstract. We discuss progress over the last year on a generalization of Mertens' second theorem to k-almost primes (products of k prime numbers). It is known that the sum of reciprocals of k-almost primes not exceeding x is a polynomial of degree k in $\log\log x$, plus an error term that tends to zero. In particular, we established a much sharper estimate for the case $k = 2$. We also partially proved a conjecture discussed last year describing the coefficients in the case of arbitrary k. We will discuss this, along with progress towards completing the conjecture and describing the asymptotic behavior of the coefficients. This is joint work with Jonathan Bayless and Jared Lichtman.

Andrew Knightly (U. of Maine) *Counting locally supercuspidal cusp forms*

Abstract. Given a finite set of primes S, and a tuple $\widehat{\sigma} = (\sigma_p)_{p\in\S}$ of supercuspidal representations σ_p of $\mathrm{GL}_2(\mathbb{Q}_p)$, let $S_k(\widehat{\sigma})$ be the space of weight k cusp forms f which are unramified away from S and for which $\pi_p = \sigma_p$ for each $p \in S$, where π is the cuspidal representation attached to f. Using the trace formula, we compute the dimension of this space under certain restrictions on the σ_p.

Debanjana Kundu (CRM) *Control Theorems for Fine Selmer Groups*

Abstract. Inspired by the work of Iwasawa on growth of class groups in $\mathbb{Z}_p$-extensions, Mazur developed an analogous theory to study the growth of Selmer groups of Abelian varieties in $\mathbb{Z}_p$-extensions. He proved what is nowadays called a "control theorem", which we describe briefly here. Let A be an Abelian variety defined over a number field F with potential good ordinary reduction at all primes above p, and let $\mathcal{L}$ be a $\mathbb{Z}_p$-extension of F. For every intermediate subextension F' of $\mathcal{L}/F$, we have natural maps

$$s_{\mathcal{L}/F'} : \mathrm{Sel}(A/F') \longrightarrow \mathrm{Sel}(A/\mathcal{L})^{\mathrm{Gal}(\mathcal{L}/F')}$$

on the Selmer groups, which are induced by the restriction maps on cohomology. Mazur's Control Theorem therefore asserts that the kernel and cokernel of the maps $s_{\mathcal{L}/F'}$ are finite and bounded independently of F'. This control theorem has subsequently been generalized to more general p-adic Lie extensions by Greenberg. We will consider variants of the control theorem for a certain subgroup of the p-primary Selmer group, called the *fine Selmer group*, which in recent years has been studied extensively. The said fine Selmer group is obtained by imposing stronger conditions at primes above p. We prove various Control Theorems for fine Selmer groups of elliptic curves in a general p-adic Lie extension, where the reduction type of the elliptic curve at primes above p may not be potentially good ordinary. We will discuss certain estimates on the $\mathbb{Z}_p$-coranks of the kernel and cokernel of the restriction maps

$$r_{\mathcal{L}/F'} : R(E/F') \longrightarrow R(E/\mathcal{L})^{\mathrm{Gal}(\mathcal{L}/F')}$$

for a p-adic Lie extension $\mathcal{L}/F$. We will specialize to three cases of p-adic Lie extensions where we can show that the kernel and cokernel of the restriction map are finite, and (under appropriate assumptions) give growth estimates for their orders. This is joint work with Meng Fai Lim.

Matilde Lalin (U. de Montréal) *Non-vanishing for cubic L-functions*

Abstract.Chowla's conjecture predicts that $L(1/2, \chi)$ does not vanish for Dirichlet L-functions associated with primitive characters χ. It was first conjectured for the case of χ quadratic. For that case, Soundararajan proved that at least 87.5% of the values $L(1/2, \chi)$ do not vanish , by calculating the first mollified moments. For cubic characters, the first moment has been calculated by Baier and Young (on $\mathbb{Q}$), by Luo (for a restricted family on $\mathbb{Q}(\sqrt{-3})$), and on function fields by David, Florea, and Lalín. In this talk we prove that there is a positive proportion of cubic Dirichlet characters for which the corresponding L-function at the central value does not vanish. We arrive at this result by computing the first mollified moment using techniques that we previously developed in our work on the

first moment of cubic L-functions, and by obtaining a sharp upper bound for the second mollified moment, building on work of Lester and Radziwiłł, Harper, and Radziwiłł- Soundararajan. Our results are on function fields, but with additional work they could be extended to number fields, assuming GRH.

David Lilienfeldt (McGill U.) *Geometric quadratic Chabauty over number fields*

Abstract. We generalize the recent method of geometric quadratic Chabauty, initiated over $\mathbb{Q}$ by Edixhoven-Lido, to general number fields. This provides a conditional bound on the number of rational points of curves of genus at least 2 defined over arbitrary number fields, which satisfy the so-called quadratic Chabauty condition. This is joint work with Pavel Coupek, Luciena Xiao and Zijian Yao.

Yongxiao Lin (EPFL) *Averages of coefficients of $GL_3 \times GL_2$ L-functions*

Abstract. Given a sequence of arithmetic function $a(n)_{n\geq 1}$, it is natural to ask how strong the error term

$$\sum_{n\leq X} a(n) - (\text{possible main term})$$

can be. We will address this problem when the $a(n)$'s are specified to coefficients of $GL_3 \times GL_2$ L-functions, and explain how it is related to cancellation of an analytically twisted sum of $GL_3 \times GL_2$ coefficients. Time permitting, some related questions will also be discussed. This is joint work with Qingfeng Sun.

Adam Logan (Carleton U. and TIMC) *Covering K3 surfaces by squares of curves*

Abstract. One way to understand the ℓ-adic representations attached to a variety is to relate them to those of curves. The first case in which it is not known whether this can be done is that of K3 surfaces, although the Kuga-Satake construction suggests that it is possible. In this talk I will describe a new construction, generalizing and adapting work of Paranjape, that proves that certain families of K3 surfaces of rank 16 are quotients of the square of a curve of genus 7. This has interesting consequences for K3 surfaces in $\mathbb{P}^4$ with 15 nodes and other families. This is joint work with Colin Ingalls and Owen Patashnick.

Reginald Lybbert (McGill U.) *Extending the de Shalit-Goren foliation on unitary Shimura varieties*

Abstract. Let p be inert in the quadratic imaginary field E, and look at a unitary Shimura variety S, of signature (n, m). The de Shalit-Goren foliation is a natural foliation of height 1, and rank m^2 in the tangent bundle of the special fiber of S. It has been shown that there is a certain Ekedahl-Oort stratum S^{fol} which is an integral manifold of this foliation. We now show that the de Shalit-Goren foliation is a subbundle of the tangent bundle for each Ekedahl-Oort stratum lying over S^{fol}.

Alexander Mangerel (CRM) *Monotone chains of Fourier coefficients of holomorphic cusp forms*

Abstract. (joint w/ O. Klurman) We will discuss the proof of a recent general result, improving upon work of Bilu, Deshouillers, Gun and Luca, a consequence of which are the following: the set of positive integers n for which $\tau(n+1) < \tau(n+2) < \tau(n+3)$ has upper density $1/6$ relative to the set of non-vanishing of τ (the Ramanujan τ function), and for $k \geq 2$ the set of n such that $0 < |\tau(n+1)| < \cdots < |\tau(n+k)|$ has relative natural density $1/k!$.

Kaisa Matomäki (U. of Turku) *Multiplicative functions in short intervals revisited*

Abstract. A few years ago Maksym Radziwill and I showed that the average of a multiplicative function in almost all very short intervals $[x, x+h]$ is close to its average on a long interval $[x, 2x]$. This result has since been utilized in many applications.

I will talk about recent work, where Radziwill and I revisit the problem and generalise our result to functions which vanish often as well as prove a power-saving upper bound for the number of exceptional intervals (i.e. we show that there are $O(X/h^{\kappa})$ exceptional $x \in [X, 2X]$).

We apply this result for instance to studying gaps between norm forms of an arbitrary number field.

Youcef Mokrani (U. de Montréal) *Generalizations of Monsky matrices for elliptic curves in Legendre form*

Abstract. Let n be squarefree positive integer. The Monsky matrix of n has the property that its kernel possesses a group isomorphism with the 2-Selmer group of the elliptic curve $y^2 = x^3 - n^2x$. This quality of Monsky matrices makes them powerful tools used to find infinite families of non-congruent numbers. In this talk, we will show how the concept of Monsky matrices can be generalized to any elliptic curve that has a Legendre form over the rationals. We will also present new results on the congruent number problem and similar questions that can be found using these matrices.

Grant Molnar (Dartmouth College) *Formal summation of divergent series*

Abstract. A "summation" is an operator akin to the map that takes a series to the limit of its partial sums. We show that naturally occuring summations may be thought of as deficient ring homomorphisms, and provide methods to canonically extend these summations.

Subramani Muthukrishnan (Indian Institute of Information Technology D& M) *On the simultaneous 3-divisibility of class numbers*

Abstract. Let $k \geq 1$ be a cube-free integer with $k \equiv 1 \pmod 9$ and $\gcd(k, 7 \cdot 571) = 1$. In this talk, we prove the existence of infinitely many triples of imaginary quadratic fields $\mathbb{Q}(\sqrt{d})$, $\mathbb{Q}(\sqrt{d+1})$ and $\mathbb{Q}(\sqrt{d+k^2})$ with $d \in \mathbb{Z}$ such that the class number of each of them is divisible by 3. This affirmatively answers a weaker version of a conjecture of Iizuka.

Marc-Hubert Nicole (CRM et Aix-Marseille) *Shimura reciprocity for Drinfeld modules with real multiplication*

Abstract. We will explain the proof of the analogues over global function fields of positive characteristic of two conjectures of H. Darmon: the algebraicity of Stark-Heegner points and a Shimura reciprocity theorem for real quadratic fields. Joint work with I. Longhi (IISc, Bangalore).

Isabella Negrini (McGill U.) *Periods of modular forms and binary quadratic forms*

Abstract. In the paper *Modular Forms with Rational Periods*, Kohnen and Zagier gave examples of cusp forms whose periods are rational and arithmetically interesting. We expose some of their work and study the possibility of a p-adic variation.

Ramon Nunes (Universidade Federal do Ceará) *Spectral reciprocity via integral representations*

Abstract. A spectral (automorphic) reciprocity formula is an identity between sums of L-values of (different) families of automorphic representations. In this talk we show progress towards a generalization for number fields of an identity by Blomer and Khan. This has applications to subconvexity and non-vanishing.

Gautier Ponsinet (Max Planck Institute) *Universal norms of p-adic Galois representations and the Fargues-Fontaine curve*

Abstract. In 1996, Coates and Greenberg computed explicitly the module of universal norms for abelian varieties over perfectoid field extensions. The computation of this module is employed in Iwasawa theory, notably to prove "control theorems" for Selmer groups, and generalizes Mazur's foundational work on the Iwasawa theory of abelian varieties over $\mathbb{Z}_p$-extensions. Coates and Greenberg raised the natural question on possible generalisations of their result to general motives. In this talk, I will present a new approach to this question relying on the classification of vector bundles over the Fargues-Fontaine curve, which allows us to answer Coate and Greenberg's question affirmatively in new cases.

Sudhir Pujahari (U. of Hong Kong) *On the normal number of prime divisors of sums of eigenvalues of Hecke operators*

Abstract. In 1917, Hardy and Ramanujan showed that any natural number has $\log\log n$ number of distinct prime factors. In 1984, R. Murty and K. Murty studied the normal number of prime factors of Fourier coefficients of modular forms. Recently, using recent developments in the theory of l-adic Galois representations, we obtained analogue results for sums of eigenvalues of Hecke operators. This is joint work with R. Murty and K. Murty.

Brent Pym (McGill U.) *Multiple zeta values in deformation quantization*

Abstract. "Deformation quantization" is a mathematical abstraction of the passage from classical to quantum physics: starting from a classical phase space (a Poisson manifold) we deform the ordinary multiplication of functions to produce a noncommutative ring, which serves as the algebra of quantum observables. A landmark 1997 theorem of Kontsevich shows that such quantizations always exist, and moreover gives an explicit formula in local coordinates.

The formula is a Feynman expansion involving volume integrals over the moduli space of marked holomorphic disks. I will describe joint work with Banks and Panzer, in which we prove that these integrals evaluate to integer-linear combinations of multiple zeta values, building on Brown and Goncharov's theory of polylogarithms on the moduli space of stable rational curves.

James Rickards (McGill U.) *Computing with (indefinite) quadratic forms and quaternion algebras in PARI/GP*

Abstract. Motivated by my thesis topic, I will describe a package I am writing in PARI/GP to deal with (primarily indefinite) quadratic forms and quaternion algebras. By pointing out how I used these algorithms in my thesis project, I also hope to convince audience members that computers are an invaluable tool in modern number theory research.

Julian Rosen (U. of Maine) *The p-adic periods of number fields*

Abstract. Consider an algebraic variety X defined over the rational numbers. For each prime p of good reduction, there is a distiguished $\mathbb{Q}_p$-linear automorphism of $H^*_{dR}(X) \otimes_{\mathbb{Q}} \mathbb{Q}_p$, the p-adic Frobenius, coming from the theory of crystalline cohomology. Matrix coefficients for this automorphism (with respect to a $\mathbb{Q}$-basis for $H^*_{dR}(X)$) are called p-adic periods. One can view these numbers as p-adic analogues of usual complex periods, which arise as matrix coefficients for the isomorphism between de Rham cohomology and Betti cohomology. In this talk, I will discuss the p-adic periods of 0-dimensional varieties (i.e. the spectra number fields).

Giovanni Rosso (Concordia U.) *Drinfeld modular forms and their families: old, new, and unknown*

Abstract. In this talk we shall introduce Drinfeld modular forms for $\mathbb{F}_p[[T]]$ from a geometric point of view, and explain some results on congruences modulo T and how one can try to construct ordinary and finite slope families. We shall conclude with some open problems in the field.

Andrew Schultz (Wellesley College) *Galois module structure of square power classes in biquadratic extensions*

Abstract. In a field K of characteristic 2, Kummer theory says that the square power classes $K^\times/K^{\times 2}$ form an $\mathbb{F}_2$-vector space which parameterizes elementary 2-abelian extensions of K. When K is Galois over a field F, then there is a natural action of $\mathrm{Gal}(K/F)$ on these power classes, and the submodules under this action gives those elementary 2-abelian extensions of K that are Galois over F. In this talk we describe the (surprisingly simple) structure of $K^\times/K^{\times 2}$ as a Galois module.

William Verreault (Université Laval) and Chenghui Zheng (U. of Toronto) *Limiting distributions of sums of random multiplicative functions over function fields*

Abstract. (Two-person presentation) We study the limiting distribution of partial sums of random multiplicative functions defined over function fields where the number of irreducible prime factors of the polynomials is restricted. This parallels work of Harper where the partial sums of ± 1 random multiplicative functions were taken over integers $n \leq x$ with $k = o(\log\log n)$ prime factors. Based on a recent formulation by Soundararajan and Zaman, we also study an analogous problem for partitions into k distinct parts.
This work is joint with Daksh Aggarwal, Unique Subedi, William Verreault, Asif Zaman, and Chenghui Zheng.

Aled Walker (CRM and Trinity College Cambridge) *Sets with large greatest common divisors, and the Duffin–Schaeffer conjecture*

Abstract. Let $A \subset \{1, \ldots, X\}$ be a set of square-free numbers, and suppose that at least 1% of the pairs $(a_1, a_2) \in A \times A$ satisfy $\gcd(a_1, a_2) \geqslant D$. Must it be the case that $|A| \ll X/D$. If so, this would be tight, considering the case in which A consists of all multiples of D. We will discuss this and other similar questions in combinatorial number theory, motivated in part by Koukoulpoulos–Maynard's proof of the Duffin–Schaeffer conjecture.

Jason Zhao (UCLA) *Determining optimal test functions for 2-level densities*

Abstract. Katz and Sarnak conjectured a correspondence between the n-level density statistics of zeros from families of L-functions with eigenvalues from random matrix ensembles, and in many cases the sums of smooth test functions, whose Fourier transforms are finitely supported over scaled zeros in a family, converge to an integral of the test function against a density $W_{n,G}$ depending on the symmetry G of the family (unitary, symplectic or orthogonal). This integral bounds the average order of vanishing at the central point of the corresponding family of L-functions.

We can obtain better estimates on this vanishing in two ways. The first is to do more number theory, and prove results for larger n and greater support; the second is to do functional analysis and obtain better test functions to minimize the resulting integrals. We pursue the latter here when $n = 2$, minimizing

$$\frac{1}{\Phi(0,0)} \int_{\mathbb{R}^2} W_{2,G}(x,y)\Phi(x,y)dxdy$$

over test functions $\Phi : \mathbb{R}^2 \to [0, \infty)$ with compactly supported Fourier transform. We study a restricted version of this optimization problem, imposing that our test functions take the form $\phi(x)\psi(y)$ for some fixed admissible $\psi(y)$ and supp $\widehat{\phi} \subseteq [-1, 1]$. Extending results from the 1-level case, namely the functional analytic arguments of Iwaniec, Luo and Sarnak and the differential equations method introduced by Freeman and Miller, we explicitly solve for the optimal ϕ for appropriately chosen fixed test function ψ. We conclude by discussing further improvements on estimates by method of iteration.

Posters and/or 20 minute video clips

Below are to be found the titles and abstracts for the posters and/or the 20 minute video clips. They are listed in alphabetic order according to the first letter of the last name of the participant submitted on the google sheet registration form. I only included the participant names for which I had received at least a title (before September 14 and for which the title was perceived as being sufficiently close to what the organizing committee understands as "Number Theory") and for which the participant had agreed to make a poster and/or a 20 minute video clip.

Résumés / Abstracts

Ambreen Ahmed (ASSMS) *Modular properties of elliptic genus of Hilbert scheme of points on complex plane*

ABSTRACT. not sent.

Saralee Aursukaree (Silpakorn U.) *On exactly* 3*-deficient-perfect numbers*

ABSTRACT. Let n and k be positive integers and $\sigma(n)$ the sum of all positive divisors of n. We call n an exactly k-deficient-perfect number with deficient divisors $d_1, d_2, \ldots, d_k$ if $d_1, d_2, \ldots, d_k$ are distinct proper divisors of n and $\sigma(n) = 2n - (d_1 + d_2 + \ldots + d_k)$. In this article, we show that the only odd exactly 3-deficient-perfect number with at most two distinct prime factors is $1521 = 3^2 \cdot 13^2$.

Ayush Bohra (Shiv Nadar U.) *Permanents of* 2×2 *matrices over* $\mathbb{Z}_n$

ABSTRACT. In this talk, we prove that $|GL_2(\mathbb{Z}_n)| = \sum_{d|n} \phi(\frac{n}{d}) \times |G_n(d)|$, where $G_n(x) = \{A \in GL_2(\mathbb{Z}_n) | perm(A) \equiv x \pmod{n}\}$. We also compute $|G_n(x)|$, for every natural number n and $x \in \mathbb{Z}_n$.

Alex Bongiovanni (Kent State University) *Representations of large integers as the sum of fractional powers of primes and squares*

ABSTRACT. We prove that all large integers can be written in the form $p + [m^c]$ for any fixed $1 < c < 69/62$, and that almost all large integers can be written in the form $[p^c] + m^2$ for any fixed $1 < c < 419/352$.

Jonathan Chapman (U. of Manchester) *Partition and density regularity for Diophantine systems*

ABSTRACT. A system of equations is called partition regular if every finite colouring of the positive integers produces monochromatic solutions to the system. A system is called density regular if it has solutions over every set of integers with positive upper density. A classical result of Rado characterises all partition regular linear systems, whilst Szemerédi's theorem classifies density regular linear systems. In this talk, I will report on recent developments in the partition and density regularity of non-linear systems of equations.

Sumit Kumar (Indian Statistical Institute Kolkata India) *Delta function and its application to subconvexity*

ABSTRACT. In this talk we will prove subconvexity bounds for the $GL(3) \times GL(2)$ Rankin-Selberg L-functions.

Joshua Males (U. of Cologne) *Self-conjugate 7-cores and class numbers*

ABSTRACT. I will present recent results obtained jointly with Kathrin Bringmann and Ben Kane. We begin by extending results of Ono-Raji to provide a formula for the number of self-conjugate 7 core partitions in terms of a single class number - the proof relying on standard techniques in modular forms. To complement this, I will also give a sketch of the underlying combinatorial structure and explanation. This involves abaci and extended t-residue diagrams of partitions and the final step relies heavily on Gauss' results in the genus theory of binary quadratic forms.

Neeraj Kumar Paul (Gauhati U.) *On Vajda's and Vajda-like identities via generalized Fibonacci and Lucas numbers*

ABSTRACT. For Fibonacci sequence $\{F_n\}$, Vajda's identity is stated as $F_{n+i}F_{n+j} - F_nF_{n+i+j} = (-1)^n F_iF_j$. Fibonacci sequence is generalized in terms of the number of sequences generated. This generalization generates m sequences and $m = 1$ gives the particular case of Fibonacci sequence. Using this generalization, Vajda's identity is

expressed in terms of determinant of order $m+1$. Catalan's and Cassini's identities follow as special cases. Lucas equivalent for these identities are also obtained.

Srinivas Thiruchinapalli (Research Scholar) *Triangular Numbers definition for all integers*

Abstract. In this paper , we are revisiting the topic of TRIANGULAR NUMBERS with new perspective direction in defining them. We know that such numbers are used in practical applications like Handshake Puzzle, Full Mesh Network, Number Strip Puzzles. Previously they were only defined for positive integers as follows: $T_n = n(n+1)/2$. Now we can extend the definition to all integers: $T_n = n(n+1)/2$ if $n \geq 0$ and $n(n-1)/2$ if $n < 0$. In this generalization, we proved some results like relation between Integers to triangular numbers, relation between formation of triangular numbers with squares and how to define binary operations addition and multiplication on triangular numbers. Also we have proved some more properties which are satisfied by Triangular numbers.

Biao Wang (State U. of New York at Buffalo) *Analogues of Alladi's formula*

Abstract. In this talk, we will mainly introduce the analogue of one of Alladi's formulas over $\mathbb{Q}$ with respect to the Dirichlet convolutions involving the Möbius function $\mu(n)$, which is related to the natural densities of sets of primes by recent work of Dawsey, Sweeting and Woo, and Kural et al. Several examples will be given. For instance, if $(k, \ell) = 1$, then

$$-\sum_{\substack{n \geq 2 \\ p(n) \equiv \ell (\operatorname{mod} k)}} \frac{\mu(n)}{\varphi(n)} = \frac{1}{\varphi(k)},$$

where $p(n)$ is the smallest prime divisor of n, and $\varphi(n)$ is Euler's totient function. This refines one of Hardy's formulas in 1921. At the end, We will give some conjectures on more analogues.

Peter Zenz (McGill U.) *On the Fourth Moment of Holomorphic Cusp Forms*

Abstract. The so-called Random Wave Conjecture predicts that the moments of holomorphic cusp forms agree with the moments of a complex gaussian random variable as the weight tends to infinity. In this talk we will investigate the fourth moment, which has a close relationship to moments of L-functions. Using techniques of Harper and Soundararajan, on obtaining sharp bounds for the moments of L-functions (under GRH), we show that the fourth moment of holomorphic cusp forms is bounded.
